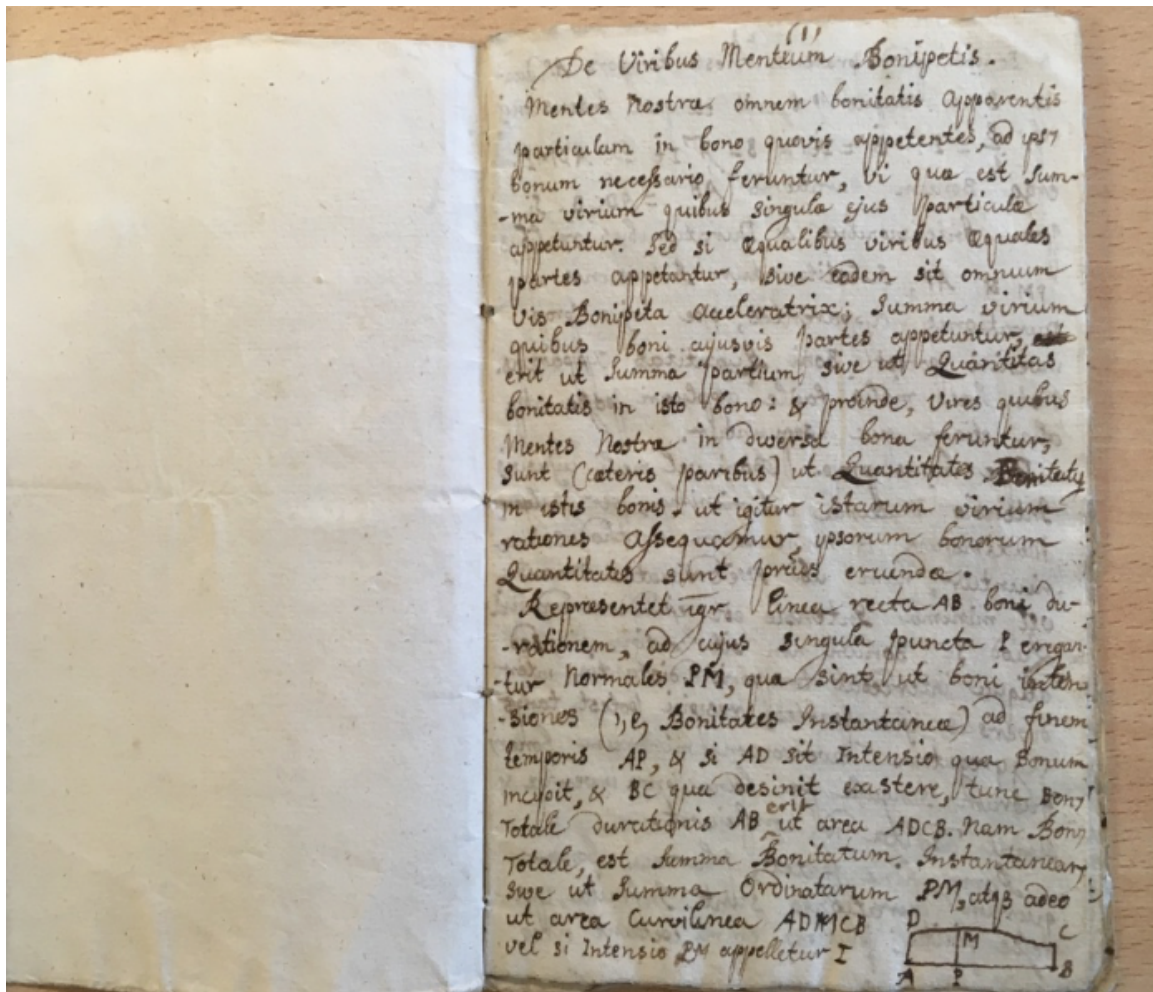


MACLAURIN AND MORALITY

David Horowitz
Golden West College (Emeritus)
Huntington Beach, CA
docmath@juno.com

Isobel Falconer
University of St. Andrews
Scotland
ijf3@st-andrews.ac.uk



Part 1. Introduction

Scottish mathematician Colin MacLaurin (1698-1746) is best remembered for his *A Treatise of Fluxions* (MacLaurin, 1742), *An Account of Sir Isaac Newton's Philosophical Discoveries* (MacLaurin, 1748), and the appellation for a type of power series. However, it is hardly known that in 1714 at the age of sixteen MacLaurin penned a short manuscript wherein he tried to apply Newtonian principles to morality. *De Viribus Mentium Bonipetis* (*On the Good-seeking Forces of Minds*) (MacLaurin, [1714]) remained unpublished and hidden in the papers of the Colin Campbell Collection for over 250 years. This handwritten manuscript was only uncovered at the end of the twentieth century. It provides a remarkable glimpse into how the young MacLaurin dealt with early Newtonianism, the tenets of the Church of Scotland, and the nascent interface between science and religion in the early eighteenth century.

This paper examines the content of *De Viribus Mentium Bonipetis* in depth for the first time, drawing on Ian Tweddle's unpublished translation (Tweddle, 2008; rev. 2018).¹ After accounts of the biographical background to, and history of, the manuscript, it will:

¹ Tweddle's manuscript is now available on the University of St. Andrews' *MacTutor* website:

- present an outline of MacLaurin's mathematical discussion in *De Viribus*;
- explain how MacLaurin's theological statements in *De Viribus* can be grounded in his mathematical observations; and
- discuss MacLaurin's rationale in his composition of *De Viribus* in the context of early eighteenth-century social and religious mores in Scotland.

Interpreting *De Viribus* in this way provides a lens for examining the intellectual context of early eighteenth-century Scotland within which such an endeavor made sense to the youthful MacLaurin, newly graduated from the University of Glasgow.

Part 2. Biographical Background

Colin MacLaurin was born in February 1698 in the manse of Kilmodan Church in Clachan of Glendaruel on the Cowal peninsula of Scotland². He was the third child (all sons) of the Reverend John MacLaurin (c.1658-1698)³,

https://mathshistory.st-andrews.ac.uk/Publications/tweddle_maclaurin.pdf. The authors would like to thank Ian Tweddle who has professionally and personally motivated this investigation.

² The old manse (now replaced) is still accessible but in ruins. Clachan of Glendaruel became known simply as Kilmodan, the religious parish in which it is situated. 'Clachan' is the Gaelic word for 'stones.' No doubt the village took its name from the ornate carved gravestones found nearby which date from the fourteenth and fifteenth centuries. Many are currently displayed in a small lapidarium located in the southwest corner of the church graveyard (Brydall, 1905, pp. 28-31; Mooney, 1994, pp. 48-49, Ross [undated]).

³ John MacLaurin is remembered for publishing an early translation of the Book of Psalms into Gaelic in 1694 (Brown, 1889, p. 371).

the minister of the local parish church, and his wife Mary Cameron (1647-1707) (Scott, 1923, p. 31). When Colin's father died six weeks after his birth, his mother relocated with her sons to Dumbarton 'for the more convenient education of her children' (MacLaurin, 1748a, p. i).⁴ On her death in 1707, Colin and his oldest brother John (the middle sibling, Daniel, having died) went to live with their uncle, the Reverend Daniel MacLaurin (c.1650-1720)⁵, in the village of Kilfinan in Argyllshire. Daniel was also a minister of the local church, and both boys had strong religious upbringings.⁶ By the middle of the seventeenth century, Scotland was almost entirely a Calvinist country, whether in the Presbyterian (today associated with the Church of Scotland) or Episcopal (aligned with the Anglican Church) form.⁷ The cyclical pattern of competing Episcopal and Presbyterian supremacies in Scotland had turned toward the latter. With the accession of William and Mary in 1688,

⁴ At the time of the MacLaurins' sojourn in Dumbarton, the minister of the local parish church was John Anderson (McIntosh, 2004). He cared for the sick, administered relief to the poor, and catechized each individual family on a regular basis (MacPhail, 1984, p. 52). To the chagrin of more conservative Presbyterian clergy, Anderson amended the church service by including the Lord's Prayer. Thus even as a child, Colin MacLaurin was influenced by a liberalization of Scottish church doctrine.

⁵ In 1707, Daniel MacLaurin was tasked by the Synod of Argyle with translating the Westminster Confession of Faith into Gaelic. The project was ultimately completed in 1725 (Brown, 1889, pp. 374-375). For more on Daniel MacLaurin and parish church of Kilfinan, see MacLaurin, 2012, pp. 410-411.

⁶ John MacLaurin (1693-1754) became a leading theologian in the Church of Scotland in Glasgow. He was 'regarded as one of the greatest Scottish preachers of his day...' (Sher, 2004).

⁷ The differences between the two branches ran deep and centered around worship (Presbyterian liturgical readings versus Episcopal adoption of the *Book of Common Prayer*), the structure of church hierarchy (local Presbyterian ministers and elders versus Episcopal bishops who derived their authority from the monarch), and observance (Presbyterian refusal to celebrate saints' days – even Christmas). (Brown, 1997, pp. 14-17) 'Thus the development of presbyterian and episcopalian cultures had ensured that protestant uniformity was an unachievable ideal in Scotland.' (Raffe, 2012, p. 50)

Presbyterians saw an opportunity to expand their control over Scottish parishes. The Revolution Settlement of 1689-90 reestablished Presbyterian dominance over the Kirk, and both father and uncle were among those Presbyterians who were hastily enlisted to fill vacant positions previously occupied by Episcopal ministers in Scotland (Harris, 2007, p. 376; Mills, 1982, p. xv; Raffe, 2018, pp. 131-156).⁸ Thus by the time of Colin MacLaurin's birth, his family was firmly established in the ecclesiastical life of Scotland.

In 1709 Colin MacLaurin matriculated at the University of Glasgow and enrolled in the four-year masters degree.⁹ At eleven years old, he was not unusually young for such advancement at the time. During this period, the university operated under the traditional regent system wherein each cohort of students was overseen by a single generalist supervisor who gave his lectures in all subjects throughout the course.¹⁰ In a slight twist that presaged a move to a specialist professorial system, the University of Glasgow had one regent, the Professor of Greek, dedicated to first year ('bejan') students; the

⁸ Colin's father, John MacLaurin, was ordained at Kilmodan parish in 1689; his Episcopal predecessor, Duncan MacArthur, had held that ministry from 1685 to 1688 when he was ousted. Similarly, in 1698 Daniel MacLaurin succeeded to the position previously occupied by Episcopal minister Robert Stewart at Kilfinan; the latter had held that post from 1675 to 1690 before also being ejected. (Scott, 1923, pp. 29, 31.)

⁹ The MA was (and is) the first degree at Scottish universities although the BSc is now awarded in sciences. In addition, some of the newer universities (e.g. University of Stirling) have begun to award the BA degree.

¹⁰ For a description of the regent educational system in Scottish universities during the seventeenth and early-eighteenth centuries see Shepherd, 1974, pp. 18-60.

bejans then were passed to another regent who taught their second and subsequent years.¹¹ MacLaurin is originally listed among fifteen first-year students who were supervised by regent Alexander Dunlop.¹² The regent for MacLaurin's final three years was the renowned Scottish philosopher Gershom Carmichael (Shepherd, 1974, p. 376).¹³ Carmichael's description of his teaching methods and duties at Glasgow gives an idea of the curriculum MacLaurin would have studied (Carmichael, 1712; reprinted in Carmichael, 2002, p. 379):

In the Magistrand [fourth] Year (if God spare them & me together), ye first work must be to compleat what yet remains undone of ye Ethicks, & then, if possible, again to glance over ye Pneumatick & Ethick Theses:¹⁴ tho' at the same time, I must endeavour, with all convenient Speed, to get them thro', at least, ye sixth Book of Pardies Elements, without which they can make verry few Steps to purpose in ye Physicks: I must likewise give them a touche of some other parts of Geometry, as time will allow; but for their more thorow acquaintance with them, refer them to ye Professor of Mathematicks.

¹¹ The curriculum during the seventeenth and eighteenth centuries remained remarkably uniform among the four Scottish universities: the first year was devoted mainly to Greek, the second to logic, the third to ethics, and the fourth to physics, with several other courses inserted into this framework (Forbes, 1983, p. 30).

¹² Alexander Dunlop (1684-1747) was appointed Professor of Greek at Glasgow in 1704 and resigned his chair in 1742 (Rigg and Lloyd, 2004). His 1709-1710 bejan student is identified as 'Colinus M^cLaurine' (University of Glasgow, 1854, p. 195).

¹³ Gershom Carmichael (1672-1729) received masters degrees from the University of Edinburgh (1691) and the University of Glasgow (1693) before being appointed regent at Glasgow in 1694. A polymath, he taught all aspects of the philosophy curriculum including Greek, logic, metaphysics, moral philosophy and natural philosophy (Carmichael, 2002, pp. 377-388). He is regarded as a principal figure in the Scottish Enlightenment (Moore and Silverthorne, 2003; Waymack, 1986, pp. 5-56).

¹⁴ Pneumatick in this context meant Pneumatology – study of the spirit – and hence related to ethics and questions of free will and natural or Divine law.

‘Pardies Elements’ was an introductory textbook on Euclidean geometry by the Jesuit priest Ignace Gaston Pardies (1636-1673) that originally was written in French (Pardies, 1671).¹⁵

Thus, the study of mathematics beyond basic geometry was an extracurricular option. For MacLaurin, ‘ye Professor of Mathematicks’ was Robert Simson, who had arrived at the University of Glasgow in 1711.¹⁶ Simson had a keen interest in the geometry of the ancient Greek mathematicians including the porisms of Euclid. Although there is no record of MacLaurin having formally studied with Simson as an undergraduate, historians frequently assume that he did.¹⁷ However, their close friendship in subsequent years inspired MacLaurin’s geometric investigations (Tweddle, 1991; Tweddle, 2000, p. 3; Wickman, 2016, p. 7).¹⁸

¹⁵ A Latin edition was published in 1694 in Oxford and may be the edition used by Carmichael since there is a copy in Glasgow University Library. English editions appeared in 1701, 1702, 1705, 1711 but there is no record of any at Glasgow ([https:// estc.bl.uk](https://estc.bl.uk)).

¹⁶ Robert Simson (1687-1768) assumed the Professorship of Mathematics at Glasgow in 1711, fully two years after MacLaurin had matriculated, and held that position for half a century (Carlyle and Tweddle, 2004)). Among his most important writings are a book on conic sections (Simson, 1735), an account of the *loci plani* of Apollonius (Simson, 1749), and a definitive edition of Euclid’s *Elements* (Simson, 1756). ‘During his nearly fifty years’ tenure of the Chair his clear and careful teaching had a far-reaching influence on the development of mathematics in Scotland.’ (Schlapp, 1973, p. 7)

¹⁷ See, for instance, Sageng, 2004; University of Glasgow, 2015. Colin MacLaurin is conspicuously absent in Lord Brougham’s list of distinguished students of Robert Simson at Glasgow (Brougham, 1845, p. 482). Perhaps this indicates that MacLaurin was not formally a student of Simson but rather that they later became friends, correspondents and colleagues.

¹⁸ In an unpublished lecture draft, Eric Gray Forbes provides strong evidence of a close, but probably informal, association between MacLaurin and Simson as early as 1712 (Forbes, 1984 & 1987). In the early 1980’s, Forbes had temporary access to one of MacLaurin’s student notebooks which was, at the time, in private hands. Forbes found a strong correlation between MacLaurin’s mathematical methods and the books known to be in Simson’s library. The student notebook was subsequently lost but has recently been rediscovered and now resides in the Glasgow University Archives and Special Collections (Horowitz, to appear).

MacLaurin received his masters degree on 23 June 1713¹⁹ for his thesis *De gravitate, aliisque viribus naturalibus* (*On gravity, and other natural forces*) (MacLaurin, 1713; printed and translated in Tweddle, 2007, pp. 6-43).²⁰ Besides discussing its physical manifestations, MacLaurin asserted in his thesis that the force and ubiquity of gravity were discernible verifications of the existence of a divine Creator.²¹ Tweddle points out that at that time it was not necessary for candidates to submit their work in published form (Tweddle, 2007, p. 11). MacLaurin had made printed copies of his thesis probably in anticipation of his application for future academic or clerical positions. Indeed, it was most likely offered in support of his successful application to his appointment as Professor of Mathematics at Marischal College, Aberdeen in 1717.

¹⁹ In the list of masters laureates conferred on 23 June 1713, MacLaurin is identified as ‘Colinus M^cLaurin’ (University of Glasgow, 1854, p. 48).

²⁰ It was around this time that individual theses replaced group disputations as a masters degree requirement at Glasgow (Russell, 1974, p. 151). The adoption of the professorial system eventually allowed individual students to read whatever subjects they desired, and, consequently, class theses were slowly phased out at the beginning of the eighteenth century (Forbes, 1983, p. 29). MacLaurin’s was one of the first of such individual theses. The Minutes of the Glasgow Faculty Meeting of 20 May 1713 show that his thesis was approved along with those of two other students (G.U.A. FA 26631):

The faculty having heard said theses found nothing in them contrary to faith & good manners and therefore allows the same to be printed, that the same candidates may have opportunity of defending them publicly Sine Praesidio in order to them obtaining the Degrees of Arts.

²¹ ‘A gravitate pendet tum hominum, tum reliquorum animantium vita & nutritio; ita ut jure meritissimo gravitatis Auctor agnoscendus sit terræ dominus & hominum conservator.’ (‘The subsistence and nutrition of humans and the other living beings depend on gravity; thus it is that the lord of the earth and preserver of mankind is to be recognized most deservedly as the creator of gravity.’) (MacLaurin, 1713, p. 4; trans. Tweddle, 2007, p. 18)

In 1713 Daniel MacLaurin sent a copy of his nephew's thesis along with a cover letter (E.U.L. MS 3099.15.4)²² to the Reverend Colin Campbell of Achnaba (1644-1726), minister of the Ardchattan and Muckairn parishes in Argyllshire, asking for the clergyman's opinion of the work:

...I send it to you, for you know some little of the Mathematicks, I beg the favour of your remarks...

Campbell was a keen astrologer (Ridder-Patrick, 2012) and also a mathematician in his own right. Like several other seventeenth-century mathematicians including Marin Mersenne (1588-1648) in France and Henry Oldenburg (c.1619-1677) in England, Campbell functioned as a sort of clearinghouse where scholars could send their work for review and dissemination (Hatch, 1998). He corresponded with many of the prominent Scottish scholars at that time including John Craig, George Cheyne, and the Gregorys (David and two Jameses) (Henry, 2004). The Church Patronage Act of 1711 (passed by the British Parliament in 1712) had recently stripped the power of appointing local ministers from heritors, church elders and members of the parochial kirk-session and restored this privilege to the traditional lay patrons (many of whom were Episcopalians) who had lost such authority at

²² The letter is dated 'Inveraray 1713'. Since it refers to Colin MacLaurin's recent fifteenth birthday 'in Feb' last', it must have been written by his uncle during March-December 1713. A typewritten transcription (no doubt created by John Eaton – see 'Part 3. Considerations of the Manuscript' below) is located in the Glasgow University Archives (G.U.A. GB MS Gen 1332). A handwritten transcription (by an unknown writer) is located in the National Library of Scotland (N.L.S. MS 3440 f. 31v).

the time of the Glorious Revolution (Warr, 1933, pp. 330-331; Chitnis, 1976, p. 51; Whitley, 2013, pp. 110-128). By writing to Campbell, Daniel MacLaurin may well have been looking out for his brilliant nephew's potential future as a minister in the Church of Scotland.²³

Campbell responded very favorably to Daniel MacLaurin in an undated letter (E.U.L. MS 3099.15.7)²⁴ admiring how the young MacLaurin...

...in the age of 15, should be so well versed in the true Principles of the Physicks, yet is but beginning to breath [sic] amongst us in these remote corners: As it is surprising in itself, so it is wonderfully acceptable to me, praying and hoping this early blossome in short time produces pleasant fruit.

Upon completing his masters degree in 1713, Colin MacLaurin began his studies in the School of Divinity at the University of Glasgow, but after a few months he withdrew and returned to live with his uncle in Kilfinan. Little is known about MacLaurin during this period. Murdoch wrote (MacLaurin, 1748a, p. ii):

In this retirement he pursued his studies with the same assiduity as he had done at the university; continuing his favourite researches in

²³ Campbell wrote to the Synod of Argyll in 1694, 'For as much as by serious and frequent prayers to Almighty God for illumination and direction, by diligent search of the Holie Scriptures, and by careful use of other ordinarie means, I am persuaded and settled in my judgment that Presbyterian [sic] Government...is so fully agreeable to the Apostolic pattern held out in the Word of God...' (N.L.S. MS 3099.1; Drummond and Bulloch, 1973, p. 8). Thus, Campbell was the ideal mathematical and ecclesiastical recipient for Daniel MacLaurin's correspondence regarding his nephew.

²⁴ This letter had to have been written sometime after Daniel MacLaurin's initial contact in 1713 but before Colin MacLaurin's reference to it in his letter to Campbell of 12 September 1714. This cited document is a typed transcription (also produced by John Eaton) of the original; it can also be found in the Glasgow University Archives (G.U.A. GB 247 MS Gen 1332).

mathematicks and philosophy, and at other times reading the best classic authors; for which he naturally had an exceeding good taste.

In 1714, shortly after leaving the university, MacLaurin penned the subject of this paper: *De Viribus Mentium Bonipetis* (*On the Good-seeking Forces of Minds*) (E.U.L. MS 3099.15.6). It is an essay in which principles of early Newtonianism are applied to the study of Christian morality.

On 12 September 1714 Colin MacLaurin wrote directly to Colin Campbell (E.U.L. MS 3099.15.1)²⁵ and enclosed his essay:

But since I have mentioned the use of Mathematics I shall beg your pardon for troubling you with some thoughts I have relating to their use in morality. I have sent them to you under the title of *De viribus mentium Bonipetis*.

This is the only reference to *De Viribus* in the letter. There is no record of Campbell's reaction to *De Viribus*, and MacLaurin never mentioned it again in any of his mathematical writings or correspondence.²⁶ His short manuscript lay dormant in the papers of Colin Campbell for nearly 250 years. Only at the end of the twentieth century was it finally discovered and recognized as an early example of the application of Newtonianism to a nonscientific subject.

²⁵ A handwritten transcription (by an unknown writer) of this letter is located in the National Library of Scotland (N.L.S. MS 3440 ff. 32-34). See also Mills, 1982, Letter [116].

²⁶ Six years later in a letter dated 6 July 1720, MacLaurin wrote to Campbell, 'I know not If [sic] you received the letter I sent you in 1714 for I have never heard from you; ...' (E.U.L. MS 3099.15.2; N.L.S. MS 3440 ff. 34-37; see also Mills, 1982, Letter [117]).

It offers a valuable insight into how science interfaced with religion in the years leading up to the Scottish Enlightenment.

Part 3. Considerations of the Manuscript

Between MacLaurin's 1714 letter to Campbell and the late-twentieth century, *De Viribus* went virtually unnoticed. It was never mentioned in any of the major biographies of MacLaurin (preface by Murdoch to MacLaurin²⁷, 1748a, pp. i-xx; Smith of the Inner Temple, [1798?], section on Colin MacLaurin [unpaginated]²⁸; Pinkerton, 1799, section on Colin MacLaurin [unpaginated]; MacLaurin, 1801, pp. xiii-xx²⁹; Wilkes, 1816, pp. 47-49; Chambers, 1835, pp. 520-530; Tweedie, 1915-1916; Gibson, 1928, pp. 79-84; Schlapp, 1949; Turnbull, 1951; McKay, 1973; Mills, 1982, pp. xv-xx), nor did it appear in commemorations of MacLaurin's birth (Hall et al., 1998) and death (Turnbull, 1951). It first seems to have been unearthed by John

²⁷ Patrick Murdoch (d. 1774) was a student of Colin MacLaurin at the University of Edinburgh. He was an excellent mathematician in his own right who wrote on conic sections, trigonometry and geographical maps. After MacLaurin's death, the manuscript *An Account of Sir Isaac Newton's Philosophical Discoveries, in Four Books. By Colin MacLaurin* was compiled by Murdoch and published for the benefit of MacLaurin's widow and children (MacLaurin, 1748a). In that work, Murdoch wrote an extensive twenty-page introduction which is the only known biography of MacLaurin written by a contemporary. See Goodwin and Du Toit, 2004.

²⁸ This draws in large part from MacLaurin, 1748a.

²⁹ This second edition of MacLaurin's *A Treatise of Fluxions* contains an eight-page biographical introduction 'Life of the Author' written anonymously by 'An Eminent Mathematician.'

Carnegie Eaton (1915-1972)³⁰ who prepared a typed (although somewhat incomplete) Latin transcription of *De Viribus* (G.U.A. MS Gen 1332), but that, too, lay dormant. It was not until the mid-1980's that *De Viribus* was finally brought to light. In his 1989 doctoral thesis, Erik Lars Sageng comments on MacLaurin's treatise (Sageng, 1989, pp. 126-127):

It is easy to smile at De Viribus Mentium Bonipetis, but the graphing of qualities had been a standard part of the scholastic curriculum since its introduction by Nicole Oresme in the mid-fourteenth century, and more sophisticated moral philosophers than MacLaurin tried their hand at the quantification of various sorts of "goods."

Sageng is suggesting that MacLaurin's graphical representation of various non-scientific qualities in *De Viribus* can be associated with the concept of *configurationes* of the French scholastic Nicole Oresme (c.1320-1382) (Roche, 1998, pp. 39-41; Clagett, 1968, pp. 14-15). In a process analogous to analytic geometry, Oresme distinguished two aspects of a quality: its intensity (*latitudo*) and its duration (*longitudo*). Plotting these on respective perpendicular axes, he arrived at a shape corresponding to the quality. In a precursor to the integral calculus, Oresme asserted that this shape represented some characteristic of the quality itself.

³⁰ Eaton was a lecturer in mathematics at Glasgow's Royal Technical College, now the University of Strathclyde. For biographical information on Eaton see McKay, 1973 and Mills, 1982, p. xi.

Since Sageng, a few others have considered *De Viribus* and offered their own insights (Maas, 2003; Grabiner, 2004, 2014 & 2021; Shulman, 2008; Constanda, 2009; Wood, 2017; Bruneau, 2005, 2011 & 2020). However, it was Ian Tweddle who first carefully analyzed MacLaurin's curious manuscript (Tweddle, 2018, p. 2):

Nevertheless, I find some interesting mathematics in [*De Viribus*] and in the rest of the enclosure and I am sure that MacLaurin would not have submitted to Campbell's scrutiny anything that he did not consider to be a worthwhile development.

Tweddle carefully translated *De Viribus* and provided extensive notes on its technical content. However, he approached *De Viribus* as a mathematical document and did not examine the historical or personal context wherein a young MacLaurin composed it. Tweddle also passed over MacLaurin's often cryptic statements about morality which reflect early eighteenth-century Scottish religious mores that certainly permeated the mathematician's life and thought. If one delves deeply into *De Viribus*, then one discovers profound aspects of the author's personal beliefs. Grabiner whimsically comments (Grabiner, 2004, p.843):

A sort of trial run of the Newtonian style was Maclaurin's youthful attempt – he was sixteen – to build a calculus-based mathematical model for ethics. In a Latin essay still (perhaps mercifully) unpublished today, “*De Viribus Mentium Bonipetis*” (“On the Good-Seeking Forces of Mind”), Maclaurin mathematically analyzed forces by which our minds are attracted to different morally good things.

Yet if there is more to Colin Maclaurin than his mathematics, then there is more to *De Viribus* than a misguided early exercise in the calculus. One can interpret it as an attempt to reconcile the moral precepts of his religion with the genius of his academic background. It provides a valuable insight into the intellectual climate within which such an endeavor made sense – an era in which the potentially applicable domain of Newtonian calculus was still fluid, and, for many, the evidence of Scripture took precedence over that of the natural world.

Part 4. *De Viribus Mentium Bonipetis* [1714]

De Viribus Mentium Bonipetis (*On the Good-seeking Forces of Minds*) comprises the first ten pages of a 17-page handwritten booklet penned by MacLaurin.³¹ The final seven pages are titled *Prop. altera*³² and present an exposition on various aspects of hydrostatics. Included in *Prop. altera* are

³¹ The only extant copy of *De Viribus Mentium Bonipetis* is in the Colin Campbell Collection (E.U.L. MS 3099.15.6). The pages of this small manuscript are handwritten in ink on both sides and measure only 3.75 inches (width) x 6 inches (height). The front (f. 1r), inside front (f. 1v), and rear (f. 10v) are blank. Its ten folios are thin, and the ink from each page sometimes ‘bleeds’ through to the other side. The first eleven written pages (ten pages of *De Viribus* and the first page of *Prop. altera*) are numbered in the format ‘(1), (2), ... (11)’ at the top middle. Pages 12 through 17 are unnumbered. The seven diagrams in *De Viribus* are drawn mostly freehand. The four diagrams in *Prop. altera* seem to have incorporated a straightedge.

³² Ian Tweddle (Tweddle, 2018, p. 17) interprets this as the abbreviation for the Latin feminine singular *Propositio altera* or ‘A Second Exposition’.

MacLaurin's proofs of two propositions offered in the *Principia* but left unproven by Newton.

MacLaurin began *De Viribus* without any motivation or preamble. He commenced in Newtonian-like style by asserting that our minds seek every small bit (*particula*) of goodness in any good thing. He then proceeded to examine various mathematical distributions of such good things, calculating the total good using fluxional calculus and offering mathematical and moral consequences for each of these. He went on to investigate various possible mathematical force laws between the *particula* and the mind, again exploring the mathematical and moral consequences. Ultimately, he settled upon a force law that best reflected his own theological beliefs and brought his manuscript to an abrupt close without any conclusions or commentary.

It should be noted that the earliest known work of Colin MacLaurin is a student notebook (only recently found) produced during 1712-14 while at Glasgow (Forbes, 1983, p. 33). In the notebook he wrote out three dictates (in Latin) presumably based upon the lectures of his regent Gershom Carmichael. One of these dictates, *Pars prima ethicae*, contains a passage:

‘...those thoughts [*cogitationes*] for which experience shows that no precedent nor mandate of intellect nor command of will is required...are the most moral among all actions...’³³

³³ For a time during the 1970-80's, the original MacLaurin notebook was in the possession of Professor Eric Gray Forbes (1933-1984) of the University of Edinburgh. He (or possibly one of his students)

MacLaurin's notebook presentation of Carmichael's teaching suggests that *De Viribus* might in part have been influenced or inspired by his regent's instruction on ethics. The idea presented in this passage is that some of man's innate and instinctive thoughts can be just as 'moral' as those thoughts that he deliberately and purposely advances. It is these inherent thoughts that one might identify with the mind's natural good-seeking force that is the subject of *De Viribus*.

The morality presented in *De Viribus* can be considered a natural extension of a theological theme that became popular in the sixteenth century in Britain and throughout Europe. 'Physico-theology' emerged as 'the aspect of natural theology that seeks to prove the existence and attributes of God from the evidence of purpose and design in the physical universe.' (Carré, 1967, p. 300)³⁴ English philosophers, clergymen and naturalists such as William Whiston, William Derham and John Ray published treatises seeking

produced typewritten transcriptions of the three dictates written by MacLaurin (E.U.L. CLX-A-213 folder 4). The translation given here is based upon this transcription:

...istarum cogitationum ad quas experientia docet nullum requiri praeivium neque intellectus dictamen, neque voluntatis imperium...inter omnes actus maxime sint morales...

³⁴ The term 'physico-theology' was first used in the title of the 1711-12 Boyle Lectures given by the English clergyman and natural philosopher William Derham (1657-1735): *Physico-Theology or, a Demonstration of the Being and Attributes of God, from the Works of his Creation*. (Derham, 1714). However its adjectival form 'physico-theological' appeared fifty-nine years earlier in Walter Charleton's (1619-1707) *The Darknes [sic] of Atheism Dispelled by the Light of Nature: A Physico-Theologicall Treatise* (Charleton, 1652).

to establish the existence of a Deity from support they found in the physical universe. While these men based their work on observations in the natural world, it is not difficult to see how MacLaurin might have expanded this by combining the strong religious beliefs that he acquired from his family and education with his expertise in mathematics. In doing this, MacLaurin drew upon the natural philosophy teachings of Gershom Carmichael. Carmichael had studied Newtonianism in 1687-1691 while a student at the University of Edinburgh. Waymack cites Carmichael's *Synopsis Theologiae Naturalis* (Carmichael, 1729, pp. 112, 14-15, 17-18) and explains (Waymack, 1986, pp. 17-18):

Carmichael does, though, appeal repeatedly to empirically based arguments for God's existence. The laws discovered by Newton, according to Carmichael, show that the material world is supported and directed by an immaterial being. There must be an immaterial being to conserve and maintain the existence of the material world. Furthermore, the orderly arrangement serves as evidence of benevolent design.

MacLaurin not only considered himself a devout Christian but also believed that mathematics both complemented and supported his religious convictions. In a document³⁵ addressed to an unknown recipient (probably written in late 1734 or early 1735), he wrote:

³⁵ A.U. MS 206.65. It is unclear whether this document was first composed as a letter responding to Bishop George Berkeley's critique of Newton's *Principia* (Berkeley, 1734), or as a draft of MacLaurin's forward for his defense of Newton's calculus, *A Treatise of Fluxions*. In 1734 MacLaurin wrote to Scottish mathematician James Stirling:

I am satisfied that the interests of true Science and true Religion are united, & that they do real prejudice to Mankind who endeavor to represent them as opposite in any measure.

During the first half of the eighteenth century, the youthful MacLaurin was not the only Scot to try to relate Newtonianism with nonscientific disciplines. George Davie writes (Davie, 2001, p. 12):

To come now to their attitude to the problem of psychology, both [George] Turnbull and Maclaurin are evidently just as much taken as [David] Hume was with the notion of introducing the method of experimental reasoning into moral subjects, and of thereby doing for the problem of the mind what Newton had done for the problem of matter.

However *De Viribus* represents a much more Newtonian and technical analysis than those of Turnbull and Hume.³⁶

MacLaurin may also have derived inspiration for *De Viribus* from the work of contemporary mathematician and fellow Scotsman John Craig (c. 1663-1731).³⁷ In the preface to his 1699 treatise *Theologiae Christianae*

Upon more consideration I did not think it best to write an Answer to Dean Berkeley but to write a treatise of fluxions which might answer the purpose and be useful to my scholars. (Mills, Letter [140], p. 250)

Murdoch later added:

[MacLaurin] began an answer to the bishop's book; but as he proceeded, so many discoveries, so many new theories and problems occurred to him, that, instead of a vindictory pamphlet, his work came out as a complete system of fluxions, with their applications to the most considerable problems in geometry and natural philosophy. (MacLaurin, 1748a, pp. vi-vii)

The first part of MacLaurin's apologia *A Treatise of Fluxions* was printed in 1737, but the complete work was not published until 1743 – fifty-six years after the *Principia*. For a discussion of MacLaurin's role in establishing and defending Newton's calculus, see Grabiner, 1997. See also Mills, 1982, Letter [204] and footnote (21), p. 425.

³⁶ Davie is referring to Hume, 1739 and Turnbull, 1740.

³⁷ Born in Dumfries, Scotland, John Craig (alt. Craige) became a vicar in the Church of England as well as a proponent of both Newtonian and Leibnizian calculus. Craig first met Newton at Cambridge in 1685. His work was mainly concerned with quadratures (Dale, 2004). He became the first in England to use

Principia Mathematica, Craig asserted (Craig, 1699, p. v; trans. Nash, 1991, p. 53)³⁸:

It seems absurd not to be able to extend the usefulness of mathematics, “the divine science,” beyond the narrow boundaries of this life. As the whole world of nature is made stable by geometric laws, how can anyone doubt that these lead us on to the knowledge of nature’s omniscient Creator?

Craig used nascent probability theory to predict that the Second Coming of Christ would not occur before the year 3144 CE (Craig, 1699; Nash, 1991; Stigler, 1986). He based his derivation on a scriptural passage from the Gospel of Luke 18:8³⁹ along with the assumption that the credibility of historical evidence diminishes with the passage of time. Although it is unlikely that MacLaurin and Craig were personally acquainted with each other at the time of the writing of *De Viribus* (Craig was living in the southern English county of Wiltshire and was some thirty-four years the elder), they later developed a close friendship. In his letter to Campbell dated 6 July 1720, MacLaurin described how he and Craig had met in London a year earlier, had become ‘intimately acquainted’, and had seen each other ‘almost every day’

Leibniz’s derivative notation (Craig, 1685, p. 28 passim; Cajori, 1921, p. 453) and integral sign (Craig, 1703, p. 1346 passim; Cajori, 1921, p. 453) in a publication.

³⁸ The original Latin is as follows:

Scientiæ enim adeo Divinæ utilitatem, non ultra angustos vitæ hujus extendi posse, absurdum plane videtur; quandoquidem per regulas Geometricas omnia Naturæ opera stabiliantur, ecquis dubitet easdem nos ducere ad sapientissimi Naturæ Autoris cognitionem?

³⁹ ‘I tell you, He will see that they get justice, and quickly. However, when the Son of Man comes, will he find faith on the earth?’ (Luke 18:8) Others who gave mathematical derivations for dates of the Second Coming based upon Scripture are John Napier (Book of Revelation 8:2) and Isaac Newton (Book of Daniel 7:25, 12:11 and 12:12). For a discussion of these and other apocalyptic predictions see Horowitz, 2018.

(E.U.L. MS 3099.15.2; N.L.S. MS 3440 ff. 34-37; Mills, 1982, Letter [117]).

Even though Craig was an early Newtonian, his apocalyptic derivation is simply algebraic. MacLaurin's *De Viribus* relies heavily on the calculus.

MacLaurin never provided an explanation of the 'good' that the forces of the mind are seeking in *De Viribus*. However, an interpretation that this 'good' refers to God's grace in the afterlife makes sense of the manuscript.⁴⁰ Indeed MacLaurin wrote on page 8 (f. 5v) of *De Viribus*, '...dum bona vitae futurae consideramus' ('...while we are considering the good things of a future life...').⁴¹ With this understanding, we can see the Newtonian parallel of the good-seeking force of the mind with the gravitational force of attraction presented in the *Principia*. In the latter, Newton used spatial distance to quantify the force of gravity resulting from a distant mass. In *De Viribus*, MacLaurin employed time as his means of defining the force of the promise of a future grace.

⁴⁰ Although the four others (Sageng, 1989; Grabiner, 2004; Tweddle, 2008 & 2018; Bruneau, 2005, 2011 & 2020) who have closely examined *De Viribus* have not adopted this interpretation, it seems to make the most sense of the mathematical and morality observations made by MacLaurin in the manuscript.

⁴¹ MacLaurin later wrote to Francis Hutcheson in 1728 (Mills, 1982, Letter [13]):

I sometimes please myself with thinking how agreeably some good men I know who do not believe a future state will be surprized [sic] to find themselves live after Death. The pleasure of finding ourselves exist after Death and the Wonderful manner of it will be an admirable Introduction into a new state...

In but a single sentence early in *De Viribus* MacLaurin mentioned the obverse notion of ‘bad things’ (*De Viribus*, p. 2, f. 2v; trans. Tweddle, 2018, p. 4):⁴²

Note: these ideas and also others which follow can easily be applied to bad things.

Similarly, he offered no explanation of the meaning of ‘bad things’. However, these may refer to moral vices that are often pursued by a man during his lifetime. It is the tension between such good (posthumous) and bad (mortal) things that sheds some light on MacLaurin’s discussions of moral conflicts found later in the manuscript.

It should be noted that at the time of MacLaurin’s writing of *De Viribus*, the general assurance of posthumous Divine grace for all of mankind was contrary to fundamental (Calvinist) doctrine wherein an individual’s eternal bliss was predetermined at birth (‘unconditional election’) and could not be changed by any contemporary behavior (Palmer, 2010, pp. 29-48). Nevertheless, ‘... [by 1712] some modification in the Calvinist hard line starts to show among orthodox writers’ (Stewart, 2010, p. 35). Prominent among those writers was the Scottish theologian William Wishart the Elder (1660-1729) who became minister of Glasgow’s Tron Church (1707-1729) and

⁴² The original Latin in *De Viribus* is as follows:

‘Not: haec malis facile applicari posse atque alia etiam quæ sequuntur.’

Principal of the University of Edinburgh (1716-1728) (Jenkins, 2004). Faculty and students at Scottish universities would have been familiar with his sermons later published in his magnum opus *Theologia* (Wisheart [sic], 1716).⁴³

Part 5. The Mathematics of *De Viribus*

MacLaurin's Newtonian model for morality is presented at the start of *De Viribus*. When describing the microscopic force of attraction of gravity, Newton considered a distant point mass separated by an intervening spatial length. In an analogous manner, MacLaurin theorized a good-seeking force of minds (*vis mentium bonipeta*) of an anticipated Divine grace separated from the present by an intervening period of time. His use of the phrase 'distance of time' (*temporis distantia*) for this period made the analogy with gravitation clear (*De Viribus*, p. 5, f. 4r). MacLaurin hypothesized two quantities (varying with time) that each contributed to this force. First,

I = the intensity of a good thing (*intensio*)

⁴³ '[Wishart's *Theologia*] shows a more philosophical cast of mind than fits the conventional stereotype of a Calvinist preacher, arguing that revealed religion, which forms the basis for human salvation, can only establish its credentials where natural religion, despite our limitations, has laid the foundations for belief.' (Stewart, 2000, p. 62)

at time t , analogous to the density of a mass at point x . Later in the manuscript he introduced

v = the good-seeking accelerating force (*vis bonipeta acceleratrix*)

where v may also vary with t and was analogous to Newton's acceleration of gravity that varies with x .⁴⁴ Although both I and v were measured from a present time $t = 0$, it was the posthumous time that contributed to the good-seeking force of minds as described by MacLaurin; death marked the near boundary of the good thing.

At the outset, MacLaurin referred to a diagram that plotted the intensity of the good thing against time (Figure 1) and used fluxional notation to derive the total good, $\mathcal{B}$, in terms of I and t :

$$\mathcal{B} = \mathcal{F}I\dot{t}$$

where the symbol $\mathcal{F}$ stood for the fluent and $\dot{t}$ was the differential of the time.

In modern integral notation,

$$\mathcal{B} = \int_{t_1}^{t_2} I(t) dt$$

where $t_1 \leq t \leq t_2$ is a posthumous time period (that may be infinite).

⁴⁴ At the time of MacLaurin's writing, the idea of a 'force' was amorphous. Concepts such as the 'impressed force', the 'accelerative force', and the 'living force' referred to quantities that would be incommensurate by modern standards. MacLaurin's 'good-seeking accelerating force' followed in (and even expanded upon) this early eighteenth-century approach. See Roche, 1998, pp. 99-100.

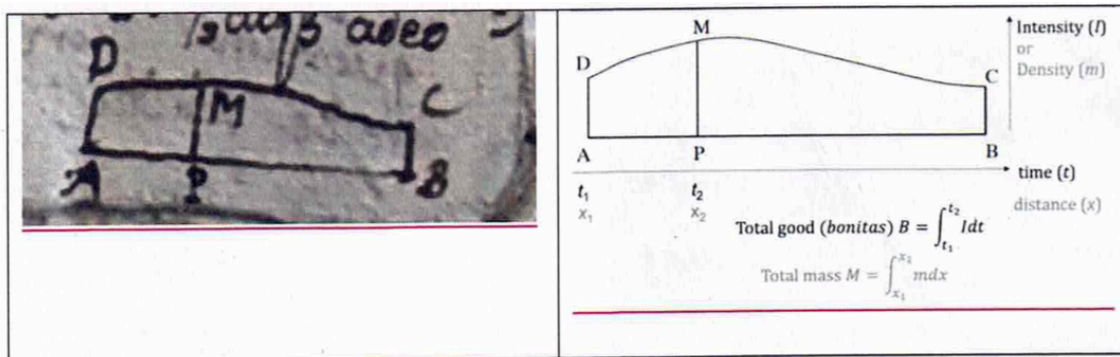


Figure 1. On the left, MacLaurin's original diagram from *De Viribus* showing how to calculate the total good; on the right, the diagram in modern form showing the analogy with calculating a (one dimensional) total gravitational mass. Time runs along the horizontal axis. The intensity of the good $I(t)$ at a given time is represented by the height of the vertical line (such as PM) drawn at that time. Thus, the total good B accumulated between the posthumous time period $t_1 \leq t \leq t_2$ is the area of ADMP.

MacLaurin next discussed the total good B arising from various distributions of the intensity $I(t)$ (Cases A, B, C in Table 1) before introducing the good-seeking accelerating force v and the possibility that this also may vary with 'distance of time' from the observer (the living person) whose mind experienced the force. He proceeded to test various intensity distributions and good-seeking accelerating force laws against his theological views, moving freely backwards and forwards between mathematics, theology, and a few observations on human behavior. In *De Viribus* MacLaurin seldom used algebraic or fluxional notation and expressed his reasoning in words and geometrical diagrams instead. An example of his

description of how to combine the intensity of a good thing and the force law is depicted in Figure 2.

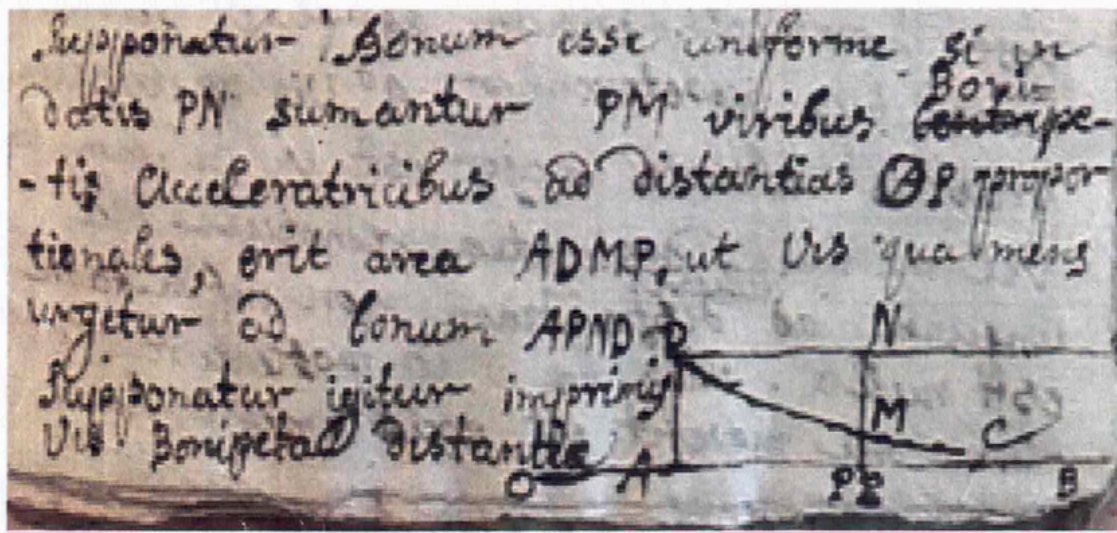


Figure 2. ‘Let it be supposed that the good thing is uniform. If in the given PN the PM are taken proportional to the good-seeking accelerating forces at the distances OP, the area ADMP will be as the force by which the mind is driven towards the good thing APND.’ (*De Viribus*, p. 5, f. 4r; trans. Tweddle, 2018, pp. 7-8.)

MacLaurin’s diagram, which shows the curve representing the force law DMC terminating at the vertex D, has been misinterpreted by commentators as taking A (rather than O) as the origin of time.⁴⁵ However, MacLaurin’s diagram clearly shows O to the left of A, and his wording makes clear that it is the ‘distance of time’ OP that standardizes the good-seeking accelerating force v . In Figure 3, which depicts MacLaurin’s two quantities in modern

⁴⁵ Tweddle acknowledges that MacLaurin’s ‘distance of time’ is ‘just the elapsed time from the start’ (Tweddle, 2018, p.13). However, he does not explicitly specify when this ‘start’ occurs. Sageng does not mention ‘distance of time’ in his discussion (Sageng, 1989).

form, we avoid this confusion by showing the good-seeking accelerating force v (commencing at a present time t_0) as clearly distinct from the intensity I of the generalized posthumous good pictured in Figure 1 (commencing at a posthumous time t_1) with which the force v is being combined.

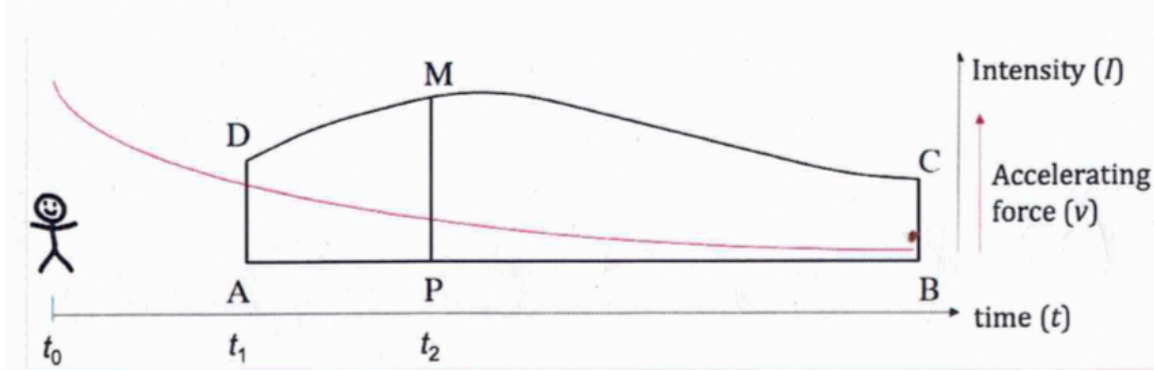


Figure 3. The intensity I of a good thing and the good-seeking accelerating force v plotted on a posthumous time period $t_1 \leq t \leq t_2$.

Thus in its general modern form,

$$\text{Good-Seeking Force of Minds} = \int_{t_1}^{t_2} I \cdot v \, dt$$

measures the total good-seeking force exerted on the living mind from a posthumous time period $t_1 \leq t \leq t_2$.

From this point, MacLaurin adhered to a generalized Newtonian method but did not adopt the inverse square form of the gravitational force law. Instead, he held the intensity I constant (except in one case) while examining various forms for v (Cases D, E, F, G in Table 1). In each case he commented on the nature of the resulting good-seeking force of minds from a

mathematical perspective and tested his results against his theological and moral beliefs in a series of statements that are often cryptic to the modern reader of *De Viribus*. They show how MacLaurin was using theology, rather than the natural world, as the testbed for the various posited forms. The interplay between the mathematics and theology/morality hidden beneath the difficult eighteenth-century Latin phraseology is present throughout the entire manuscript.

Table 1 shows, in modern notation,⁴⁶ the various functional forms of I and v that MacLaurin considered in *De Viribus*.

⁴⁶ In *De Viribus*, MacLaurin used Latin terminology such as *progressio arithmetica* (arithmetic progression), *progressio geometrica* (geometric progression), *arithmetice proportionalis* (arithmetically proportional), and *reciproce proportionalis* (reciprocally proportional) to refer to these functions.

Table 1. FORMS OF QUANTITIES in *De Viribus*
(the letters in the left-hand column are the authors' and used solely for reference)

Our Reference (in the order presented in <i>De Viribus</i>)	$I(t)$ Intensity of a Good Thing	$v(t)$ Good-Seeking Accelerating Force
A	$I = I_0 a^{-\beta t} \quad (\beta > 0)$ ‘the intensity decreases in a geometric progression while the times increase in an arithmetic progression’	$v = 1$ v not stated but later described as assumed constant
B	$I = k(t - a)^\beta \quad (\beta > 0)$ ‘the intensities of the good thing increase uniformly taking their beginning from nothing’	$v = 1$ v not stated but later described as assumed constant
C	$I = k$ ‘uniform good things whose intensities are variable’	$v = 1$ v not stated but later described as assumed constant
D	$I = 1$ ‘the good thing is uniform’	$v = \frac{k}{t}$ ‘let the good-seeking force be supposed to be reciprocally proportional to the distance of time’

E	$I = k_1 t$ ‘good things which are as the distances of the times of the mind from them’	$v = \frac{k_2}{t}$ ‘let the good-seeking force be supposed to be reciprocally proportional to the distance of time’
F	$I = 1$ I not stated but assumed constant	$v = k a^{-\beta t} \quad (\beta > 0)$ ‘the good-seeking accelerating force decreases in a geometric progression while the times increase in an arithmetic progression’
G	$I = 1$ I not stated but assumed constant	$v = \frac{\left(2\sqrt{a} - \frac{t}{\sqrt{a}}\right)^2}{4}$ $(0 \leq t \leq 2a)$ ‘let $\dot{v}$ be as $v^{1/2}$ or $\sqrt{v}$’

At least one of the two quantities I and v was held constant in all but one of the cases (Case E), and this allowed MacLaurin to consider the total good-seeking force of minds to be proportional to either the total goodness (*bonitas*) of God's grace

$$B = \int I dt$$

or to the total natural force of attraction (*vis*) of the mind which we (but not MacLaurin) denote by $\mathcal{V}$

$$\mathcal{V} = \int v dt$$

Both are given here in modern (integral) notation. It seems likely that MacLaurin did not know, in general, how to work out the result in cases where both I and v varied.

Case E is an exception which led to a simple result. In pursuing the consequences of a force law v that is 'reciprocally proportional to the distance of time', MacLaurin pointed out in Cor. 3^o that if the intensity I were allowed to vary in direct proportion to the distance of time, then the total force remains constant whenever it is observed (*De Viribus*, p. 6, f. 4v; trans. Tweddle, 2018, p. 8):⁴⁷

⁴⁷ The original Latin in *De Viribus* is as follows:

'v.g. Bonum quodvis hora ab hac distans ejusque duplum duas post horas obtinendum æqualiter secundum hanc hyp. appeterentur.'

For example, any good thing separated from the present by one hour and its double to be taken after two hours would be sought equally according to this hypothesis.

This is simply stating that

$$(I \cdot v)|_{t=1} = k_1 1 \cdot \frac{k_2}{1} = k_1 k_2 = k_1 2 \cdot \frac{k_2}{2} = (I \cdot v)|_{t=2}$$

However, a force law v that is ‘reciprocally proportional to the distance of time’ runs into problems when one extrapolates into the present $t = 0$. As t approaches zero, the force rises rapidly; in modern terms it is undefined at $t = 0$. MacLaurin realized this in Cor. 4^o but did not quite know what to make of the undefined quantity written in modern notation $(I \cdot v)|_{t=0} = \left(k_1 t \cdot \frac{k_2}{t}\right)|_{t=0}$. He decided to interpret this as an infinite force.⁴⁸ In doing so, he asserted that this quantity was ‘infinitely greater’ than $(I \cdot v)|_{t=A}$ for any time $A > 0$ ⁴⁹

The force into the present good thing is infinitely greater than the force into the same good thing removed even by a very small distance. And never at any non-infinite distance is it nothing. (*De Viribus*, p. 6, f. 4v; trans. Tweddle, 2018, p. 7)

⁴⁸ MacLaurin was not the only young man to be baffled by the presence of zeros in a fraction. As a student at Cambridge, even Isaac Newton observed in his handwritten notebook *Questiones quædam Philosophiæ* (Newton, c.1664-c.1665, f. 89r; reprinted in McGuire and Tamny, 1983, p. 347):

Thus $\frac{2}{0}$ is double to $\frac{1}{0}$ & $\frac{0}{1}$ is double to $\frac{0}{2}$, for multiply the 2 first by 0 & divide the 2^{ds} by 0, & there results $\frac{2}{1} \cdot \frac{1}{1}$ & $\frac{1}{1} \cdot \frac{1}{2}$. yet if $\frac{2}{0}$ & $\frac{1}{0}$ have respect to 1 they beare the same relationship to it that is $1 : \frac{2}{0} :: 1 : \frac{1}{0}$. & ought therefore to be considered equall in respect of a unite.

While MacLaurin’s discourse in *De Viribus* is arguably more palatable than Newton’s penned fifty years earlier, both are analytically troubling to the modern mathematician.

⁴⁹The original Latin in *De Viribus* is as follows:

‘Vis in bonum præsens infinité major est vi in idem bonum vel minima distantia remotum. Nec unquam ad ullam distantiam non infinitam est nulla.’

He concluded his analysis of Case E with the following unexplained statement (*De Viribus*, pp. 6-7, ff. 4v-5r; trans. Tweddle, 2018, p. 7):⁵⁰

And these last Corollaries [3° and 4°] trouble me to the extent that I am of the opinion that this hypothesis is not true.

What bothered MacLaurin so much about corollaries 3° and 4° that made him decide to discard Case E? He gave no rationale, and the interested reader can only conjecture.

It may be that MacLaurin's troubles emanated from his finding 'infinite' moral quantities as consequences of his mathematical analyses. Corollary 3° may have prompted MacLaurin to reject Case E since the constant $k_1 k_2 \neq 0$ meant that he could not compute the total good-seeking force of minds over an unbounded posthumous time beyond A . Similarly, the 'infinite' nature of $(I \cdot v)|_{t=0}$ in Corollary 4° implied an infinite force into the present good that would have been beyond mortal experience. Alternatively, MacLaurin may have been interpreting Corollary 4° in the context of 'bad things': an infinite force into a present bad thing would overwhelm any good-seeking force at that time and preclude all mortal human virtue.

⁵⁰ The original Latin in *De Viribus* is as follows:

'Atque haec duo posteriora Corollaria me movent ut existimem hanc hyp. non esse veram.'

Whatever the nature of his unstated objections, in composing *De Viribus* MacLaurin also had to honor Christian dogma handed down from Augustine and Aquinas that Scriptural passages bore the absolute power of certainty (Harrison, 2007, p. 110). Around 1540, the Austrian-born mathematician Georg Joachim Rheticus (1514-1574) wrote (Hooykaas, 1984, p. 65):

For it is written that one shall not diverge from the words of the Lord, either to the right or to the left, and that the Word itself has the force of demonstration, since it has been given to us by God.

Around the same time, John Knox (c.1514-1572) carried this ideology to Scotland (Kyle, 1992, p.65):

But for the most part, Knox made God's Word, as revealed in Scripture, his sole authority. In questions pertaining to the faith, Knox subordinated all other authorities, whether they be the Church of Rome, church councils, tradition, individual conscience, majority opinion, or even princes and parliaments to God's Word.

Any deviation from established Scriptural principles had the potential for serious consequences.⁵¹ MacLaurin was no doubt aware of the fate of another student, Thomas Aikenhead (d. 1697) of the University of Edinburgh, who was tried, convicted and hanged after allegedly transgressing Scottish

⁵¹ By the 1690's, the Kirk's anxiety over its waning authority over Scottish secular life had become clear. (Hunter, 1995, p. 322) In 1698, the English Parliament passed 'An act for the more effectual suppressing of Blasphemy and Profaneness' which authorized imprisonment, financial penalties, and even death for 'whoever hereafter shall in their writing or discourse, deny, impugn or quarrel, argue or reason, against the being of God, or any of the persons of the blessed trinity, or *the authority of the Holy Scriptures of the Old and New Testaments*, or the providence of God in the government of the world' (emphasis added). (Raithby, 1820, p. 409; Webb, 1992, p. 162)

blasphemy laws.⁵² In addition, while at Glasgow MacLaurin had witnessed firsthand the charges of heresy leveled at Professor of Divinity John Simson beginning in 1712, and he knew that *De Viribus* would be welcomed by the Reverend Colin Campbell only if it were consistent with contemporary religious mores. In post-Reformation Scotland, the Bible had replaced the authority of the Catholic Church as the sole basis of the Christian faith (Johnston, 1962, pp. 251-252). Even at the beginning of the eighteenth century, the absolute authority of Scripture continued to serve as a theological testbed for science.⁵³

Earlier in *De Viribus*, MacLaurin did much better in dealing with the undefined and the infinite. When introducing his discussion of the total good $\mathcal{B}$, he offered examples of curves of infinite length that bound a finite area with their asymptotes. Among them were the cissoid of Diocles as well as higher-order hyperbolas.⁵⁴ Examples like these offered MacLaurin the possibility of an infinite afterlife while not precluding a finite total good $\mathcal{B}$, a

⁵² Among other offenses, it was asserted that Aikenhead ‘endeavored to ridicule the holy scriptures, calling the Old Testament Ezra’s fables, by a profane allusion to Esop’s fables, and saying that Ezra was the inventor thereof [sic]...’ (Howell, 1816, p. 919). Aikenhead was the last person in Britain (and possibly the first person in Scotland) to be executed for blasphemy. See Drummund and Bulloch, 1973, pp. 14-15; Hunter, 2004; Ahnert, 2007, pp. 19-20; Graham, M. 2013, p. 6.

⁵³ The chief accusation directed at John Simson was that he considered human reasoning to be on a par with Scriptural proclamation (Skoczylas, 2001, pp. 103-113). In a letter composed in 1711 Simson wrote, ‘It is true the Scriptures are to be believed only because the Word of GOD; but none can be obliged to believe them to be GOD’s Word without Proof...’ (Simson, 1716, p. 4)

⁵⁴ While the quadratures of the higher-order hyperbolas were well known, the corresponding result for the cissoid was only derived by Christiaan Huygens in 1658. See van Maanan, 1991, pp. 8-10.

finite total natural force of attraction of the mind $\mathcal{V}$, and a finite total good-seeking force of minds. Thus, MacLaurin could remain faithful to the Apostles' Creed that promised 'life everlasting' and which had been used in all Christian liturgies since the eighth century including Scottish Presbyterianism (Lochman, 1999, pp. 109-110).

In Case A, wherein the intensity function decreases geometrically, MacLaurin referred to a diagram (Figure 4) and used a geometric method that can be traced to Torricelli around 1644 for finding the area between a decreasing geometric curve and its asymptote (Loria, 1900, pp. 80-89; Curtis, 1978, p. 900):⁵⁵

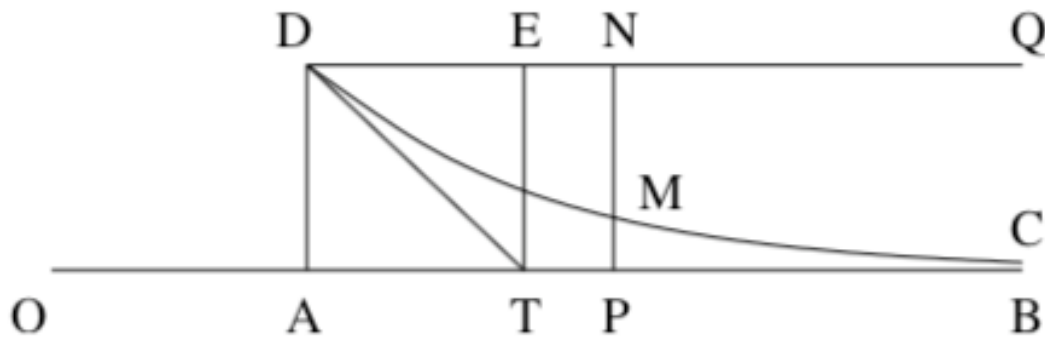


Figure 4. The unbounded area between the decreasing geometric curve DMC and its asymptote OAB is equal to the area of the rectangle ATED where the line DT is tangent to the curve at point D.

⁵⁵ This quadrature formula also appears (without proof) in a work of Christiaan Huygens (Huygens, 1690, p. 178):

Que l'espace infini entre ordonnée, la Logistique, & son asymptote, du côté que ces deux dernières vont en s'approchant, est double du triangle que font l'ordonnée, la tangente menée du mesme point que ordonnée, & la soutangente.

This book could be found in the personal library of Robert Simson (G.U.A. Sp Coll Ea5-d.8). The formula (along with a geometric proof) also appears in Grandi, 1701, pp. 16-17, 32, 85.

...and the infinite area ADCB = ATED, where it is supposed that DT touches the curve at A. Therefore a good thing of this type of infinite duration AB is equal to a uniform good thing of duration AT and of constant intensity AD. (*De Viribus*, p. 3, f. 3r; trans. Tweddle, 2018, p. 5)⁵⁶

In modern notation for decreasing geometric curves, the intensity function $I = I_0 a^{-\beta t}$ is represented by the curve DMC. Then for any time $t = a$, the point D has coordinates $(a, I_0 a^{-\beta a})$, and it is possible to define a point T = $(a + \frac{1}{\beta}, 0)$ which is the intersection of the horizontal asymptote and the tangent line to the curve DMC at the point D. It follows that the unbounded area beneath the geometric curve is finite:

$$ADCB = \int_a^{\infty} I_0 e^{-\beta t} dt = \frac{1}{\beta} \times I_0 e^{-\beta a} = AT \times AD = ATED.$$

In *De Viribus*, MacLaurin presented this technique for finding the unbounded area without proof.⁵⁷ Later when considering Case F (where the good-seeking

⁵⁶ The original Latin in *De Viribus* is as follows:

‘& area infinita ADCB = ATED, supp. quod DT tangit curvam in A. Bonum igitur hujusmodi infinitæ durationis AB æquale est uniformi durationis AT, intensionis AD constantis.’

⁵⁷ The quadrature formula appears on page 77 (CONSECTARY I.) of *A Treatise of Fluxions* by Charles Hayes (Anderson and Guicciardini, 2004) published in 1704. The Glasgow University Library obtained a copy of this book that same year, and it was also part of Robert Simson’s personal library. Murdoch claimed that Simson was one of those who encouraged MacLaurin’s academic pursuits by ‘opening their libraries to him’ (MacLaurin, 1748a, p. ii). Thus, Hayes’s book would be a likely place for MacLaurin to discover the formula (either through Carmichael, Simson, or by reading it himself). In this way, the formula’s journey from Torricelli to *De Viribus* would be complete. It can be shown that this quadrature method will only work for geometrically decreasing curves. Torricelli called these *hemihyperbola* or *logarithmica* curves (Loria, 1900, p. 80; Curtis, 1978, p. 900). The latter term should not be confused with modern logarithmic curves.

accelerating force v decreases geometrically), MacLaurin applied the same quadrature method and offered the following moral reflection (*De Viribus*, p. 8, f. 5v; trans. Tweddle, 2018, p. 8):⁵⁸

And hence comes forth a means for explaining a remarkable phenomenon: namely, that very many people who seem to be of the opinion that virtue brings an infinite good thing after itself, nevertheless pursue vices.

MacLaurin may have been observing that the good-seeking accelerating force v in Case F extending over an unbounded posthumous time interval would have produced only a finite natural force of attraction of the mind $\mathcal{V}$. Consequently, for some this might easily have been outweighed by the attraction of vices (bad things) in this lifetime.

The final (and geometrically most interesting) form that MacLaurin considered is Case G. He desired a good-seeking accelerating force v that would vanish at a finite time; this does not happen in any of his preceding six cases. To do this, he first initialized $v(0) = a$ for a fixed value $a > 0$, after which he suggested (*De Viribus*, p. 9, f. 6r; trans. Tweddle, 2018, p. 9):⁵⁹

Therefore let us see what other power of the good-seeking force we may select to which we set its decrease proportional.

⁵⁸ The original Latin in *De Viribus* is as follows:

‘Atque hinc prodiit modus solvendi phaenomenon insigne: $\overline{nim}$ quod plurimi qui virtutem bonum infinitum post se trahere opinare videntur, vitia tamen sequuntur.’

⁵⁹ The original Latin in *De Viribus* is as follows:

‘Videamus igitur quam aliam vis bonipetæ potentiam eligamus cui proportionalem ejus decrementum statuamus.’

In modern notation, Maclaurin wished the good-seeking accelerating force v to satisfy

$$\frac{dv}{dt} = -cv^m$$

for some power m (where $c > 0$ is a fixed constant of proportionality). He proceeded to give his reasons why he ruled out values where $m \leq 0$ (since v would increase and thus not vanish at any finite time), $m = 1$ (since this would reduce to Case F which he had already discarded), and $m > 1$ (since this would result in an infinite good-seeking accelerating force into some present good thing which was dismissed in Case E). Thus, MacLaurin was left to pick m such that $0 < m < 1$; he seems to have arbitrarily chosen $m = \frac{1}{2}$ (perhaps because it was at the midpoint of this interval). He ultimately settled upon the quadratic solution

$$v(t) = \frac{\left(2\sqrt{a} - \frac{t}{\sqrt{a}}\right)^2}{4}$$

for $0 \leq t \leq 2a$.⁶⁰ In Figure 5, this is represented by the curve EMB
(*De Viribus*, p. 10, f. 6).

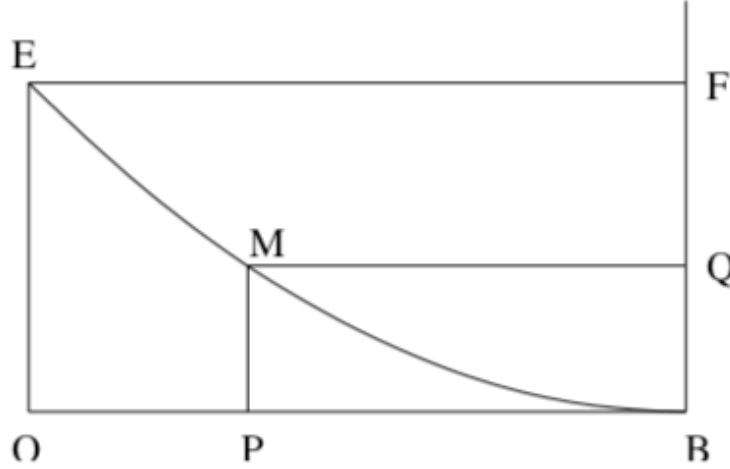


Figure 5. For a parabolic curve EMB with vertex B and P a point between O and B, the area of OEMP is equal to $(OB^3 - BP^3)/12 OE$.

MacLaurin then used the geometric properties of conics to show that the area OEMP bounded by this curve is

$$OEMP = \frac{OB^3 - BP^3}{12 OE}$$

the final statement in the essay. MacLaurin had determined the total natural force of attraction of the mind $\mathcal{V}$ for Case G on the interval $[0, t]$:

$$\mathcal{V} = \int_0^t v \, d\tau = \int_0^t \frac{1}{4a} (2a - \tau)^2 \, d\tau = \frac{(2a)^3 - (2a - t)^3}{12a}$$

⁶⁰ This placed the vertex of the parabolic curve at $(2a, 0)$. Other than being aesthetically pleasing, it is not clear why the time at which v vanished should be double the magnitude of the initial condition $v(0) = a$.

which is the modern equivalent of the final line written in *De Viribus*.

Part 6. Morality in *De Viribus*

Intermingled in *De Viribus* are several obscure passages offering what appear to be moral or theological observations. In order to make sense of these, one must (i) connect them to the underlying mathematics that they represent, and (ii) determine what theological principles might motivate their inclusion. By doing so, the reader is offered a unique glimpse into MacLaurin's personal theology as well as the morality of the Scottish society in which he lived.

In Case C, MacLaurin held both the good-seeking accelerating force v and the intensity I constant:

$$I = k .$$

After some straightforward mathematical comments, he then offered the following observation (*De Viribus*, p. 4, f. 3v; trans. Tweddle, 2018, p. 6):⁶¹

It is infinitely more preferable to increase the intensity of an infinitely lasting good thing of this type by the smallest possible quantity than to extend its duration as far as possible; for by adding DL to the intensity

⁶¹ The original Latin in *De Viribus* is as follows:

'infinité magis præstat hujusmodi infinité durantis boni intensionem vel maximé exigua quantitate augere quam ejus durationem quam maximé prolongare; Nam addendo DL intensionem AD boni infiniti ADCB, augmentum infinitum DLOC ei adjungimus; addendo autem AQ durationi infinita AB fit incrementum finiti Boni AQHD.'

AD of the infinite good thing ADCB, we adjoin the infinite increase DLOC to it; but by adding AQ to the infinite duration AB the increment of the finite good thing AQHD results.

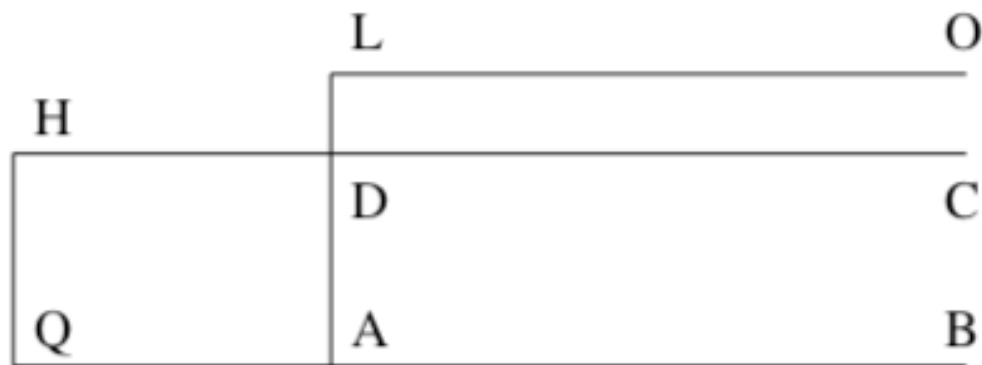


Figure 6. MacLaurin contended that the infinite area ADCB is affected less by the attachment of the finite area AQHD than by the inclusion of the infinite area DLOC.

MacLaurin's assertion here is that joining the finite area AQHD to the infinite region ADBC does not have as much effect as appending the infinite strip DLOC (regardless of how small the positive increment DL might be). Certainly at the time of his writing *De Viribus*, MacLaurin did not have our modern grasp of the infinite to comprehend that the inclusion of either area would have no effect on the infinite area of ADBC.

It is at this point that MacLaurin penned perhaps the most difficult passage in *De Viribus* both to translate and to understand (*De Viribus*, pp. 4-5, ff. 3v-4r; trans. Sageng, 1989, p. 125):⁶²

⁶² The original Latin in *De Viribus* is as follows:

Therefore the complaints by good men about the miseries of this life ought to be taken away by a consideration of the increment to the intensity of future eternal happiness which arises therefrom, whereby it may perchance happen that their whole future happiness taken together may be greater than if the starting point is assumed at the beginning of their existence, and man had never fallen. That the intensity is increased seems to be sufficiently clear from the fact that gratitude will be greater and that very many virtues will be exercised for which there would be scarcely any place if all men lived their lives as innocents without any evils or misfortunes.

Sageng's interpretative diagram (Figure 7) is helpful here:

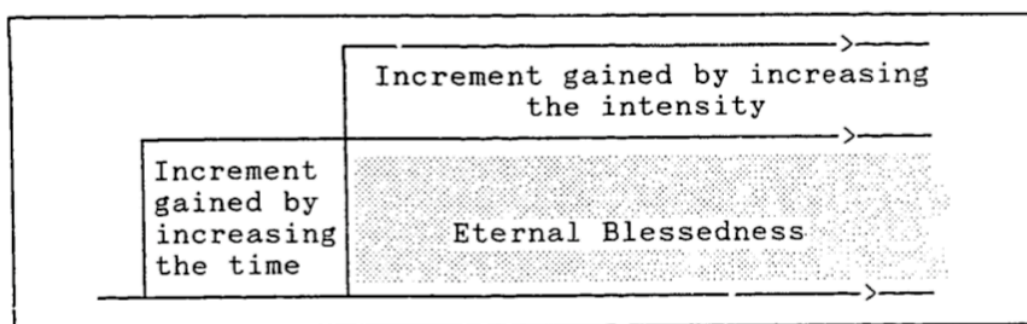


Figure 7. Sageng explains, 'If the intensity of a good is uniform and its duration is infinite, we can see by this analysis that an increase in intensity will cause an infinite increase in the total good, while an increase in duration will only cause a finite one' (Sageng, 1989, p. 125).

In other words, had man never fallen as a result of Original Sin, then God's grace as represented by the horizontal line HDC in Figure 6 would have remained constant. In that case the goodness experienced during a man's

'Bonorum igitur virorum de miseriis hujus vitæ querelas debent, perimere consideratio incrementi quod exinde intensioni fælicitatis futuræ æternæ accedit, quo (forte) fit ut eorum fælicitas tota simul sumpta major sit quam si ab initio eorum existentie exordium sumpsisset, & nunquam cecidisset homo. Intensionem autem augeri ex eo satis patere videtur quod major sit Gratitude & quod quamplurimæ virtutes exerceantur quibus vix ullus esset locus si omnes homines Innocentes sine ullis malis aut infortuniis vitam agerent.'

lifetime would be the finite area AQHD. However because of The Fall, the mortal grace experienced by any individual is less than this area. MacLaurin argued that any partial increase in eternal (posthumous) grace as represented by DLOC would be infinite (and thus outstrip any decrease in the area of AQHD) regardless of the size of the increase in intensity DL. That increment DL could only occur if man had fallen and was therefore capable of effecting such an increase in the intensity. Otherwise, the intensity of God's grace would remain constant throughout eternity.⁶³

Of course, the modern concept of mathematical infinity says that the unbounded area of ADCB is not increased by supplementing the incremental area DLOC. However, consideration of the infinite during the seventeenth and eighteenth centuries was often in the context of religion or morality (as is the case here in *De Viribus*).⁶⁴ ‘...the comparison between performing

⁶³ A similar thought is offered by Craig in Chapter 6. On Finite and Infinite Pleasures Compared to One Another (De voluptate finita & infinita inter se comparatis) of his *Theologiæ Christianæ Principia Mathematica* (1699). There he states in a corollary (Craig, 1699, p. 34; trans. Nash, 1991, p. 81):

Valor voluptatis à Christo promissæ est infinitè maior valore voluptatis vitæ præsentis. Nam Voluptas à Christo promissa est intensitatis non decrescentis, & durationis infinitæ, ut ex ipsius Historia constat voluptas vitæ præsentis est tantum intensitatis finitæ & durationis etiam finitæ, ut omnibus notum. Ergo Quantitas Voluptatis à Christo promissæ est infinitè maior Quantitate Voluptatis Vitæ præsentis...

The value of the pleasure promised by Christ is infinitely greater than the value of the pleasure of our present life. For the pleasure promised by Christ is of nondecreasing intensity and of infinite duration as is established by his own history, but the pleasure of our present life is of finite intensity and also of finite duration, as is known to all. Therefore, the quantity of the pleasure promised by Christ is infinitely greater than the quantity of the pleasure of our present life...

⁶⁴ For a historical analysis of mathematical infinity in the seventeenth and eighteenth centuries, see Nachtomly and Winegar, 2018.

operations involving mathematical infinities, and grasping religious infinities, was one that would reappear in [Newton's] writings' (Iliffe, 2017, p. 101).⁶⁵

MacLaurin offered two methods by which man might achieve such an incremental increase in posthumous Divine grace because of The Fall: (a) man's gratitude would be greater, and (b) man would exercise additional virtues. However, the implication of (b) that man's virtue could alter the nature of his afterlife ran contrary to fundamentalist (Calvinist) Presbyterian doctrine wherein a man's eternal bliss was predetermined from birth ('unconditional election') and could not be changed by any contemporary behavior (Horton, 2011, p. 15; Palmer, 2010, pp. 29-48.) Indeed the *Westminster Confession of Faith*, the canonical doctrine of the Presbyterian Church of Scotland drawn up in 1647, explicitly states (*The Humble Advice of the Assembly of Divines*, 1647, VI. 4) :

⁶⁵ Around 1664 Newton compared certain Divine attributes to the concept of infinity (Newton, c.1664, f. 12r; reprinted in Hall and Hall, 1962, pp. 102-103, 135-136):

Sed video quid metuit Cartesius, nempe si spatium poneret infinitum, Deum forte constitueret propter infinitatis perfectionem. At nullo modo, nam infinitas non est perfectio nisi quatenus perfectionibus trabuitur. Infinitas intellectus, potentiae, faelicitatis &c est summa perfectio; infinitas ignorantiae, impotentiae, miseriae &c summa imperfectio; et infinitas extensionis talis est perfectio qualis est extendi.

But I see what Descartes feared, namely that if he should consider space infinite, it would perhaps become God because of the perfection of infinity. But by no means, for infinity is not perfection except when it is an attribute of perfect things. Infinity of intellect, power, happiness and so forth is the height of perfection; but infinity of ignorance, impotence, wretchedness and so on is the height of imperfection; and infinity of extension is so far perfect as that which is extended.

Also see Janiak, 2014, pp. 26-58.

From this original corruption, whereby wee [sic] are utterly indisposed, dis-abled [sic], and made opposite to all good, and wholly inclined to all evill [sic], do proceed all actuall [sic] transgressions.

The unconditional outcome of posthumous fate was a consequence of Original Sin and The Fall of Man. That postlapsarian divine grace could not be altered was a difficult pill to swallow for scholars and clerics at the dawn of the Scottish Enlightenment.⁶⁶ Indeed one of the revolutionary new cornerstones of early eighteenth-century religious thought in Scotland was ‘a distinctive emphasis on the importance of virtuous practice over orthodoxy in doctrine’ (Maurer, 2020, p. 175). ‘This is the change of emphasis in much of eighteenth-century thought from theological doctrine to moral conduct as the true measure of religious belief’ (Ahnert, 2014, p. 2). A young MacLaurin probably inherited this liberal view of his religion⁶⁷ from his uncle with whom he lived after the death of his parents and during his composition of *De Viribus*. MacLaurin’s approach to God’s grace in the above passage is indicative of the extent of the evolution of Church of Scotland theology.

⁶⁶ The traditional, orthodox Augustinian concept of Original Sin is described in Couenhoven, 2005. For a discussion of how the concept of Original Sin evolved in early-eighteenth century Scotland see Fergusson, 2007 and Ahnert, 2014, esp. pp. 27-30. For a comparison of the moral philosophies before and after the Enlightenment see Maurer, 2020. A more general treatment of how theology developed in Scotland is given in Hazlett, 1993.

⁶⁷ Henry Reay Sefton places Colin MacLaurin’s older brother, John, in a class of liberal Presbyterian ministers known as ‘moderates’. (Sefton, 1962, pp. 2-3). This was a group that ‘absorbed rationalism from the Enlightenment, and which despised “enthusiasm” and excess in religion.’ (Brown, 1997, p. 19)

Another enlightened Scottish academic of the early-eighteenth century who struggled with the concept of The Fall was Francis Hutcheson (1694-1746). A Presbyterian born in Ulster, Hutcheson was a contemporary of MacLaurin while attending the University of Glasgow during c.1710-1717.⁶⁸ He later succeeded Carmichael in the Chair of Moral Philosophy at Glasgow from 1729 to 1746. Along with Henry Home, David Hume, Thomas Reid, Adam Smith and others, Hutcheson became representative of the ‘science of mind’ or ‘culture of mind’ moral philosophy that distinguished writers belonging to the Scottish Enlightenment period (Graham G., 2001, p. 273; Ahnert, 2014, pp. 51-55). Such philosophy attributed a natural capacity for man to distinguish between moral and immoral qualities without depending upon Scriptural dogma. In a letter to Hutcheson written in 1728, MacLaurin ruminated on him as ‘my old acquaintance’ and their commonality of thought regarding moral philosophy (N.R.A. RH1/2/497; Mills, 1982, Letter [13]). It is unknown whether Hutcheson was aware of *De Viribus*, but it is interesting to note that ten years later in 1725 he would write (Hutcheson, 1725, pp. 177-178):

⁶⁸ Hutcheson was four years older than MacLaurin but arrived at the University of Glasgow one year later. While at preparatory school in Ireland, he had completed all the required courses for the first three years at Glasgow. Consequently, he matriculated the university as a fourth year (‘magistrand’) student in 1711 and received his masters degree in 1712 (McCosh, 1875, p. 50; Moore, 2008). After spending an additional year reading classics, Hutcheson began his studies in theology (Scott, 1900, p. 15). It is therefore likely that in 1713 both he and MacLaurin attended the same lectures given by Professor of Divinity John Simson (see below). MacLaurin and Hutcheson died in the same year (1746).

The applying of a *mathematical Calculation* to *moral Subjects*, may appear perhaps at first *extravagant* and *wild*; but some Corollarys [sic], which are easily and certainly deduc'd below, may shew the Conveniency of this Attempt, if it could be further pursu'd. (emphasis Hutcheson's)

MacLaurin's above reflections in Case C of *De Viribus* parallel Hutcheson's thoughts in a subsequent essay, 'The Affections, Duty, and Worship, to be exercised toward the Deity' (Hutcheson, 1755, p. 216; Fergusson, 2007, p. 57)⁶⁹:

The mistakes, imperfections, provocations, calumnies, injuries, or ingratitude of others we shall look upon as matters presented to us by providence for the exercise of the virtues God has endued us with, by which we may more approve our selves [sic] to his penetrating eye, and to the inward sense of our own hearts, than by the easier offices of virtue where it has nothing to discourage or oppose it.

Such shared views on Original Sin most likely can be traced to the fact that MacLaurin and Hutcheson attended the same lectures at Glasgow during 1713-14 given by the controversial John Simson (1668?-1740), Professor of Divinity during 1708-1729 (Scott, 1900, p.15).⁷⁰

[Simson] was supposed to be doubtful about punishment from original sin and to have believed in Free-Will and the possibility of the salvation of the heathen. It is certain that Hutcheson was largely influenced by

⁶⁹ Although this passage appears in a compilation of Hutcheson's writing published posthumously by his son in 1755, it was composed and privately circulated prior to 1738. These essays were based upon Hutcheson's early morning 'public' (more basic) lectures in moral philosophy given in English (Turco, 2007, p. x.). This would place the composition of this passage between 1730 and 1738. Hutcheson was the first professor to use English in his lectures at Glasgow rather than the commonly used Latin.

⁷⁰ Ironically (through his half-sister, Agnes Simson) John Simson was the half-uncle of Robert Simson, MacLaurin's mathematics mentor.
(< <https://www.geni.com/people/AgnesSimson/6000000019301478692>>)

these views; for when he himself became a preacher, some of his hearers objected to traces of these heresies in his sermons.

In fact, Simson endorsed Calvinist principles but advocated a rational approach in their interpretation (Skoczylas, 2001, p. 342)⁷¹:

Simson's religion was simple, biblically based, and rational. Human reason had a divine purpose in establishing the truth of revelation, in helping Christians to understand the scriptures, and in *assisting them to accept divine grace* and struggle towards repentance and assurance. (emphasis added)

Throughout his tenure, Simson was regularly accused by conservative members of the Kirk of teaching the denial of the concept of utter depravity that was considered a consequence of The Fall. In his early writings, Hutcheson concurred with Simson that virtue was a natural human condition; this sentiment is reflected in the good-seeking force modeled in *De Viribus*.

Another cryptic theological observation in *De Viribus* occurs in the middle of his analysis of Case F where the good-seeking accelerating force is a decreasing geometric function

$$v = ka^{-\beta t} \quad (\beta > 0)$$

(given here in modern notation). MacLaurin paused to comment (*De Viribus*, p. 7, f. 5r; trans. Tweddle, 2018, p. 8):⁷²

⁷¹ The definitive biography of Simson is *Mr. Simson's Knotty Case: Divinity, Politics, and Due Process in Early Eighteenth-Century Scotland* (Skoczylas, 2001).

⁷² The original Latin in *De Viribus* is as follows:

‘Quae hyp. malitiæ ac diligentiae Temptatoris est satis consentanea, qui verisimiliter majorem adhibet operam ad vim cohibendam quo major est;’

This hypothesis is quite in accordance with the malice and diligence of the Tempter, who probably applies greater effort to the containment of force the greater it is.

MacLaurin certainly understood that the form of v found in Case F is akin to the general solution to the differential equation (given here in modern notation)

$$\frac{dv}{dt} = -\beta v.$$

Heuristically, this ODE says that a greater good-seeking accelerating force v is manifested by a greater change (negative slope) in v . This is consistent with the Christian attitude about the nature of The Tempter⁷³: a greater good-seeking force elicits a greater effort by the devil to diminish that force.⁷⁴

The language of MacLaurin's statement regarding the Tempter mirrors a passage written twenty-five years earlier by the English Presbyterian minister William Bates (1625-1699)⁷⁵:

[The Tempter's] advantage is chiefly from our security. It is easy to surprise a suspectless enemy. St. Peter straitly warns us, "be sober, be

⁷³ 'The Tempter' is one of at least twenty ways by which the devil is referred to in Christian theology. This word (Greek: ὁ πειράζων) appears at only two places in the New Testament: Matthew 4:3 and 1 Thessalonians 3:5. (Torrey, 1897, p. 272) See also 'Titles and names of the devil', [undated], <<https://www.biblestudytools.com/concordances/torreys-topical-textbook/titles-and-names-of-the-devil.html>>.

⁷⁴ In the Biblical narrative of the three Temptations of Christ (Luke 4:1-13, Matthew 4:1-11), the lure of each temptation becomes stronger with each of Jesus's rejections.

⁷⁵ William Bates was a devout Presbyterian (Wright, 2004). He studied divinity at Cambridge University and received a BA degree in 1645 and an MA degree in 1648. In 1652 he was appointed to his first ministerial position as vicar at St. Dunstan-in-the-West (London). However, he was 'ejected' after failing to subscribe to principles in the Act of Uniformity 1662. Nevertheless, Bates spent much of his life seeking (unsuccessfully) to effect the 'comprehension' of moderate Presbyterian ministers in the Anglican Church. (Marsh, 1830, pp. 317-322) His teachings included four precepts that Bates derived regarding the nature of Divine grace (Bates, 1700, pp. 166-171). Also see Beeke, 2015.

vigilant; because your adversary the devil, as a roaring lion, walketh about seeking whom he may devour.” 1 Pet. 5. 8. *His diligence is equal to his malice* (Bates, 1815, p. 122). (emphasis added)

It is interesting to speculate about how much MacLaurin’s views on morality in *De Viribus* may have been inspired by the works of Bates. Glasgow regent John Law’s metaphysics dictate of 1699-1700 referred to Bates (Shepherd, 1974, p. 152), and this might have been passed down to MacLaurin through Gershom Carmichael. As a prominent English Presbyterian minister, Bates would certainly have been well known to the Kirk. Indeed by the end of the seventeenth century, Bates’s most famous work, *The Harmony of the Divine Attributes*, had already been published in four editions (1674, 1675, 1688 and 1697). A copy of Bates’s collected works (Bates, 1700) was held by the Glasgow University Library in 1701 (G.U.A. Sp Coll Bi6-b.10).⁷⁶ It also would most likely have occupied a place in Daniel MacLaurin’s library. The younger MacLaurin’s familiarity with it would thus seem probable.

MacLaurin’s observations on mathematics and morality in Case F point to an interesting theological juxtaposition of scientific thought and attitudes

⁷⁶ This copy has a manuscript note on the title page which reads ‘Ex libris Bibliothecae Universitatis Glasguensis emptus proprijs sumptibus 1701’ (‘From the books of the Library of the University of Glasgow purchased at his own expense 1701’). See < <https://eleanor.lib.gla.ac.uk/record=b1602309> >. The authors wish to greatly thank Robert MacLean and Fiona Neale of the Glasgow University Archives & Special Collections for their assistance and bringing this (and other materials) to their attention.

toward Satan in Scotland at the turn of the eighteenth century. During the seventeenth century the concept of Satan became not just a theological construct but also an inherent part of secular life (Brock, 2011, p.26):

...the devil was not simply an evil, non-human “Other”; for early modern Scots, the demonic represented something innate and intimate about humanity itself, serving as a constant reminder of the moral depravity of man, the potential for God’s wrath, and the insecurity of salvation.

With the approach of the Scottish Enlightenment, purely theological concepts began to lend themselves to more liberal philosophical interpretations (Fergusson, 2007, p. 3):

The Reformed orthodoxy of the seventeenth century was gradually being replaced, at least in some sections of the Kirk, by a milder, more latitudinarian theology that conceded to a greater degree of autonomy to philosophical thought while simultaneously turning away from the internecine doctrinal controversies of the 17th century, particularly the struggle against Arminianism.⁷⁷

Part 7. *Prop. altera*

⁷⁷ Arminianism was a moderate religious movement that sought to temper some of the harsh precepts of traditional Presbyterianism, particularly those of unconditional election and free will. (Mullan, 1997, p. 160; Skoczylas, 2001, p. 371) Named for one of its initial proponents, Dutch theologian Jacobus Arminius (1560-1609), Arminianism quickly drew the overt wrath of staunch Calvinists during the seventeenth century. (Bangs, 1987) However, in Scotland its message of theological pluralism gradually came to be embraced by Episcopalians, Anglicans and more progressive members of the Church of Scotland. (Raffe, 2010, pp. 573-574)

The final seven pages in the manuscript containing *De Viribus* are titled *Prop. altera* and present MacLaurin's derivations of certain principles in hydrostatics. MacLaurin described two 'forces': a 'centripetal force'⁷⁸ c and a 'density' D both of which varied with the depth x below the surface of a liquid. As in *De Viribus*, MacLaurin examined various forms of these two quantities and their effects on the 'compressing force' F which he defined by

$$\dot{F} = \frac{-Dc\dot{x}}{ab}$$

(where a and b are specified constants) along with two normalizing conditions. MacLaurin subsequently identified the compressing force F and the density D , i.e., $F \equiv D$.⁷⁹ A chart comparing the three analogous 'force' quantities used in the *Principia*⁸⁰, *De Viribus* and *Prop. altera* is given in Table 2.

⁷⁸ The 'centripetal force' in *Prop. altera* is understood as a static center-seeking force consistent with its Latin root (*centrum* = center + *petere* = to seek) and not in the context of circular motion. The analogous term (*vis centripeta*) was used by Newton in 1684 in his *De Motu Corporum* (Whiteside, 1974, vol. vi, p. 34). For a discussion of how Newton used various interpretations of centripetal force in the *Principia* see Newton, 1999, pp. 102-106.

⁷⁹ While this makes no qualitative sense in terms of modern terminology, the concept of a 'force' at the beginning of the eighteenth century was imprecise and covered many diverse types of quantities. 'Power', 'pressure', and 'force' were terms that were sometimes used interchangeably (Roche, 1983, p. 100).

⁸⁰ In Book III, Propositions 1-8 of the *Principia* Newton presented his law of universal gravitation without any algebraic symbolism. In Proposition 7, Newton wrote:

Gravitatem in corpora universa fieri, eamque proportionalem esse quantitati materiæ in singulis.
(Newton, 1687, p. 411) (emphasis added)
Gravity exists in all bodies universally and is proportional to the quantity of matter in each.
(Newton, 1999, p. 810)

The classic algebraic expression of Newton's Second Law, $F = ma$, was developed long afterward. In that context, the entries in the left-hand column of Table 2 would be represented by the following algebraic symbols:

proportionalis = a
quantitas materiæ = m

gravitas = dF.

Table 2.
CORRESPONDENCE OF QUANTITIES BETWEEN *Principia*,
De Viribus AND *Prop. altera*

Newton's <i>Principia</i>	MacLaurin's <i>De Viribus</i>	MacLaurin's <i>Prop. altera</i>
PROPORTIONALITY (GRAVITATIONAL ACCELERATION) <i>proportionalis</i>	GOOD-SEEKING ACCELERATING FORCE v <i>vis</i>	CENTRIPETAL FORCE c <i>vis centripeta</i>
QUANTITY OF MATTER <i>quantitas materiæ</i>	INTENSITY OF A GOOD THING I <i>intensitas</i>	DENSITY D <i>densitas</i>
GRAVITATIONAL FORCE <i>gravitas</i>	GOOD-SEEKING FORCE OF MINDS <i>vis mentium bonipeta</i>	COMPRESSING FORCE F <i>vis</i>
	$\overbrace{(\text{GOOD - SEEKING FORCE OF MINDS})}^{\cdot}$ $= I v \dot{t}$	$\dot{F} = \frac{-Dc\dot{x}}{ab}$

In §IV Ex. II. (*Prop. altera*, p. 14 (unnumbered), f. 8v), MacLaurin examined the case when $c = \frac{ae}{e+x}$ where e is a constant.⁸¹ By the ‘method of fluents’, he derived that

$$F = D = \frac{e^2}{e + x}.$$

He then noted that if the distances $\{e + x_i\}$ were in continuous (geometric) proportion, then the corresponding centripetal forces $\{c\}$ and the densities $\{D\}$ would also be in continuous proportion. This was, in fact, Newton’s assertion in Proposition 21 of Book II of the *Principia* that MacLaurin sought to prove.⁸²

Similarly in §V Corol. III. (*Prop. altera*, p. 16 (unnumbered), f. 9v), MacLaurin examined the case where $c = \frac{ae^2}{(e+x)^2}$ and derived (as part of a more general formula) that

$$F = D = e \exp \left[\frac{-x}{e + x} \right].$$

He noted that in this case if the distances $\{e + x_i\}$ were taken in harmonic (musical) proportion, i.e.

⁸¹ Note that in MacLaurin’s notation, the letter e is not Euler’s number $e = \exp(1)$; the latter notation was not introduced until 1731 (O’Connor and Robertson, 2001).

⁸² *Principia* Book II, Proposition 21: ‘Let the density of a certain fluid be proportional to the compression and let its parts be drawn downward by a gravity inversely proportional to the squares of their distances from the center; I say that if the distances are taken continually proportional, the densities of the fluid at these distances will also be continually proportional.’ (trans. Cohen and Whitman in Newton, 1999, p. 692)

$$\frac{1}{e + x_{i+1}} - \frac{1}{e + x_i} = \text{constant},$$

then the corresponding densities $\{D\}$ would be in geometric proportion. This was, in fact, the assertion in Proposition 22 of Book II of the *Principia*.⁸³

Part 8. Observations and Conclusions

In an address at the University of Aberdeen on the occasion of the bicentenary of the death of Colin MacLaurin, Herbert Westren Turnbull, Regius Professor of Mathematics at the University of St. Andrews, declared, ‘...though the [*Principia*] had been before the world for thirty years, it was still a closed book to all but a handful of the acutest scholars’ (Turnbull, 1951, p. 1). Turnbull boldly asserted that by the early-eighteenth century only three people had truly mastered the *Principia*: David Gregory, Roger Cotes (who had helped Newton prepare the second edition) and MacLaurin (Turnbull, 1947, pp. 318-319). In a similar manner, David B. Wilson suggests that the leading Scottish Newtonians at the dawn of the Scottish Enlightenment were

⁸³ *Principia* Book II, Proposition 22: ‘Let the density of a certain fluid be proportional to the compression and let its parts be drawn downward by a centripetal force inversely proportional to their distances from the center; I say that if the distances are taken in a harmonic progression, the densities of the fluid at these distances will be in a geometric progression.’ (trans. Cohen and Whitman in Newton, 1999, p. 694)

David Gregory, John Keill, David Hume and MacLaurin (Wilson, 2009, pp. 33-68). Three of these were mathematicians of whom MacLaurin was by far the youngest. While one might wish to add a few others to these lists, the *Principia* remained beyond the grasp of most Scottish students at the time MacLaurin was composing *De Viribus* (Eagles, 1977, p. 223):⁸⁴

Scottish students of the seventeenth century were rarely more (and often less) than fourteen years of age when they entered university. Few, even by their fourth and final year, would have developed sufficient mathematical maturity to tackle [the *Principia*].

Thus MacLaurin was in a class with few others in Scotland.⁸⁵

In addition, it has been suggested that the lack of proofs in the *Principia* often stimulated scholars to expand upon its content (Greenberg, 1996, p.390):

Some researchers who penetrated the *Principia* enough to discover its limits went on to expand NEWTON's ideas beyond his own work, and they extended the scope of NEWTON's world system in important ways by this means.

⁸⁴ It has often been claimed that with the arrival of David Gregory (1659-1708) in 1683, Edinburgh became the first university in the world to hear lectures about Newtonian mechanics (Guerrini, 2004). However, such lectures were only for a few special students and not part of the general academic curriculum. 'It was not until the turn of the [eighteenth] century that Edinburgh regents began to base their lecture notes fully on Newtonian ideas.' (Eagles, 1977, p. 224). At the University of Glasgow, Newtonianism did not enter the curriculum until slightly before MacLaurin's arrival. In his regent theses of 1707, Gerschom Carmichael became one of the first at Glasgow to deal with Newtonian science as applied to gravity, light and composition of matter. (G.U.A. Sp Coll Mu3-c.3; translated in Carmichael, 2002, pp. 233-246, esp. pp. 244-246) 'Glasgow regents were clearly not converted to Newtonian ideas in the seventeenth century to the extent that Edinburgh regents were... However, somewhere between the beginning of the eighteenth century and about 1712 Newtonian philosophy obviously became accepted in Glasgow, and not just in Carmichael's teaching.' (Shepherd, 1982, p. 76; see also Fry, 2015, p. 364)

⁸⁵ In this context it should be noted that the first English translation of the *Principia* by Andrew Motte was not published until 1729, two years after Newton's death (Newton, 1729).

Indeed, a young MacLaurin may have been one of those researchers.⁸⁶ His verification in *Prop. altera* of Newton's Propositions 21 & 22 in Book II of the *Principia* (given there without proofs) could have inspired his formulation of *De Viribus*. In May 1719 during a trip to London, he met 'the great Sir Isaac Newton with whom I was very often' (Mills, 1982, Letter [117]), and in 1721 MacLaurin wrote to Campbell (E.U.L. MS 3099.15.3)⁸⁷:

And yet after all this great Author [Newton] has done it is certain he has left Light more a mysterie than ever it was thought to be; for he has discovered many strange properties of it that were not suspected before which are unaccountable by all the Laws of Mechanism. And thus the more we advance our knowledge in Philosophy the more we find our ignorance: new parts unthought off [sic] rise to our view and surprise us who expected to have found out perhaps the limits of what seems to be endless.

The quantities that MacLaurin used in *Prop. altera* (centripetal force, density, compressing force) are terminologically the same as those in the *Principia*. After considering them in his analysis, MacLaurin might have hypothesized parallel concepts for morality in *De Viribus* (good-seeking accelerating force, intensity, good-seeking force of minds).

⁸⁶ 'There were clear tendencies, especially following the widespread reception of Newton's *Principia*, toward the mathematization of all fields of knowledge from natural philosophy through medicine to moral philosophy. These tendencies manifested in various forms of taking mathematics as the epistemic ideal to be adopted in any given field.' (Demeter and Schliesser, 2019, p. 3461)

⁸⁷ A handwritten transcription (by an unknown writer) of this letter is in the National Library of Scotland (N.L.S. MS 3440 ff. 38-40). See also Mills, 1982, Letter [6].

There are notational and stylistic indications which suggest that MacLaurin's composition of *Prop. altera* likely preceded that of *De Viribus*. Early in *Prop. altera* (p. 12 [unnumbered], f. 7v; Tweddle, 2018, p. 18), MacLaurin used a script $\mathcal{F}$ and explained in an asterisked footnote at the bottom of the page that this notation was meant to denote the fluent.⁸⁸ However he used the same notation in *De Viribus* (p. 2, f. 2v; Tweddle, 2018, p. 4) and assumed that the reader was already familiar with its meaning. In addition, MacLaurin referred to *Prop. altera* in his cover letter to Colin Campbell as 'this which I have sent was the readiest for sending I have'; he offered to further apply his fluxional approach to more of the *Principia* if that would indeed be of interest to Campbell (E.U.L. MS 3099.15.1; Mills, 1982, Letter [116]). Finally, the physical forces explicitly examined in *Prop. altera* would more naturally suggest their extensions to the forces of morality in *De Viribus* than vice versa.

One might wonder why MacLaurin would place *De Viribus* before *Prop. altera* in the manuscript if the latter were composed first.⁸⁹ A possible explanation is that he was preparing the document for Campbell who was

⁸⁸ '* not. $\mathcal{F}$ hic denotat Fluentum Quantitatem' ('* note here $\mathcal{F}$ denotes the fluent quantity')

⁸⁹ *De Viribus* and *Prop. altera* are written as a ternion (6 folios) and a duernion (4 folios), respectively, and are sewn together (without a cover) in five places at the booklet's spine. The pagination that abuts the top middle edges (only pages 1-11 are numbered) looks to have been squeezed in later after the pages were written and compiled. All of the folios of *De Viribus* (pages 1 thru 10) are numbered, but only the recto side of the first folio of *Prop. altera* (page 11) is so denoted.

foremost a minister in the Church of Scotland. MacLaurin may have felt that Campbell would personally be more impressed by the morality in *De Viribus* than the hydrostatics in *Prop. altera* and wanted to make sure that *De Viribus* would be noticed by the cleric. In 1714 MacLaurin probably had not abandoned hope of becoming a minister, and Campbell would have been a good resource in that regard.⁹⁰ Moreover, a church career could lead to a university appointment along with its accompanying social and academic status (Chitnis, 1976, p. 46).

Thus from a general perspective, *De Viribus* offers an interesting intellectual-historical example: it shows how the context of the Scottish society in which MacLaurin lived affected the thought processes and motivations of the author. It also serves as a precursor to the evolution of religion in the Scottish Enlightenment. ‘Enlightenment religion involved the application of secular ideas to religious questions, and although secularization was not intended, it was implicit’ (Ahnert, 2014, p. 13). While these types of historical analyses are plentiful in the biological and physical sciences (see, for example, Kuhn, 2012; Dear, 2019), the history of mathematics often focuses on the technical and methodological at the expense of the personal

⁹⁰ ‘A degree [in the early-eighteenth century] was not yet the key to a career; even intending ministers did not need to graduate in Arts or in Divinity.’ (Brown and Moss, 1996, p. 17) It was not until five years later that MacLaurin would be offered his first academic position at Marischal College, Aberdeen.

and sociological. The efforts of a handful of hardworking individuals in unearthing *De Viribus* point to unique opportunities to analyze mathematical histories from exciting new perspectives.

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SS. Th. Professoris Primarii; necnon Amplissimi Senatus Academici consensu, & Celeberrimæ Facultatis Artium Decreto: Pro Gradu Magisterii, Summisque in Philosophia & Artibus liberalibus Privilegiis & Honoribus rite ac legitime consequendis, In Auditorio publico Academiae Glasguensis, Ad diem Junii hora post meridiem, Propugnabit Colinus McLaurin, Scotus. (Edinburgh: Apud Robertum Freebairn, Typographum Regium, 1713).

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